Spherical Couette Flow

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SOLAR SYSTEM SCHOOL

The System

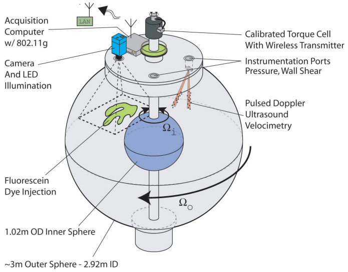


Figure taken from `http://complex.umd.edu/research/MHD_dynamos/3m.php`

- Spherical shell filled with a viscous fluid
- Flows driven by viscous torques
- Inner Boundary: Ω ; Outer Boundary: $\Omega + \Delta\Omega$
- Setup for second-generation dynamo experiments (e.g: the 3-metre experiment)

The Model

Non-dimensional parameters:

- Outer boundary rotation rate: $\Omega = \dfrac{\Omega_o L^2}{\nu} = \dfrac{1}{E}$
- Differential rotation: $\Delta\Omega = \dfrac{(\Omega_o - \Omega_i)L^2}{\nu}$
- Aspect ratio: $a = r_i/r_o = 0.35$ (Earth-like)
- Rossby Number: $Ro = \Delta\Omega/\Omega$

The Model

Scaling:

Length $\rightarrow L = r_o - r_i$, Time $\rightarrow L^2/\nu$
Mag. field $\rightarrow (\rho\mu\eta\Omega)^{1/2}$

$$\frac{\partial \mathbf{U}}{\partial t} = -\mathbf{U}\cdot\nabla\mathbf{U} - 2\Omega\hat{\mathbf{z}}\times\mathbf{U} - \nabla p + \nabla^2\mathbf{U} + \frac{1}{Pm}(\nabla\times\mathbf{B})\times\mathbf{B} \quad (1)$$

$$\nabla\cdot\mathbf{U} = 0 \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{U}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B} \quad (3)$$

$$\nabla\cdot\mathbf{B} = 0 \quad (4)$$

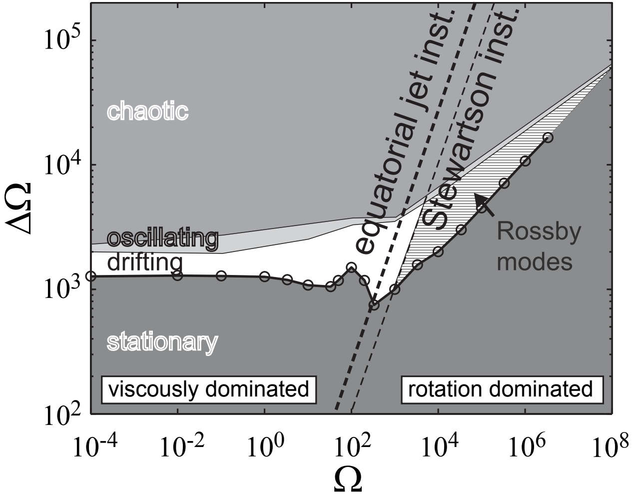
Numerics

Pseudo-spectral methods used to solve the equations in a spherical shell:

Poloidal/toroidal decomposition followed by spectral expansion (Chebyshev polynomials in radial direction, spherical harmonics in angular direction)

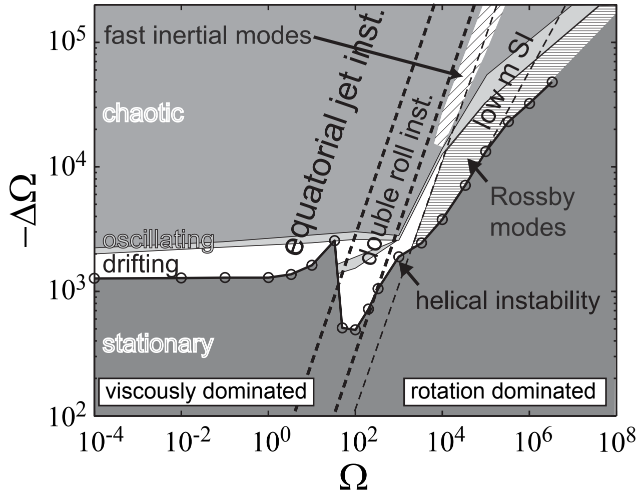
Code: MagIC (Christensen and Wicht, 2007) - Hybrid parallelization (MPI + OpenMP). Benchmarked for Boussinesq and anelastic 3D dynamo simulations.

Overview $\Delta\Omega > 0$



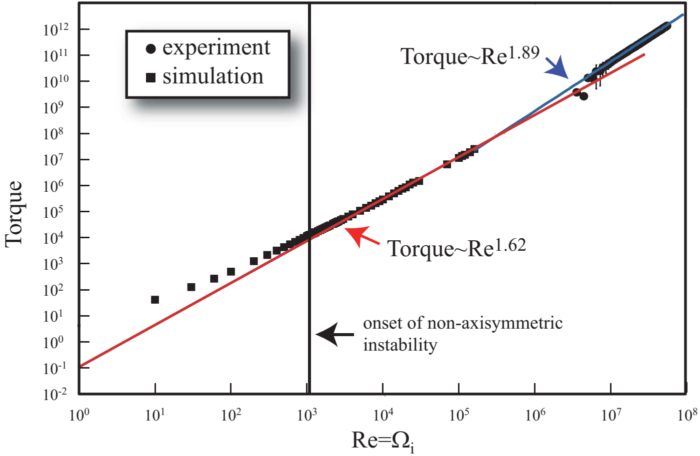
(Wicht, 2014)

Overview $\Delta\Omega < 0$



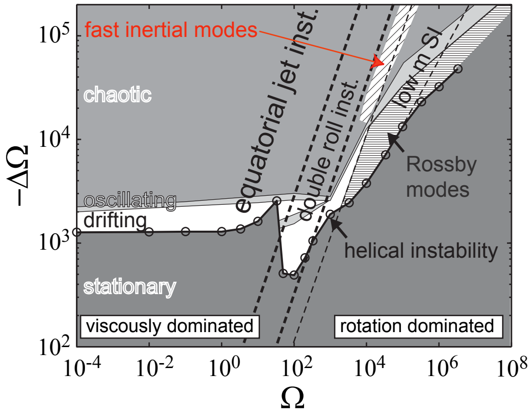
(Wicht, 2014)

Torque scaling



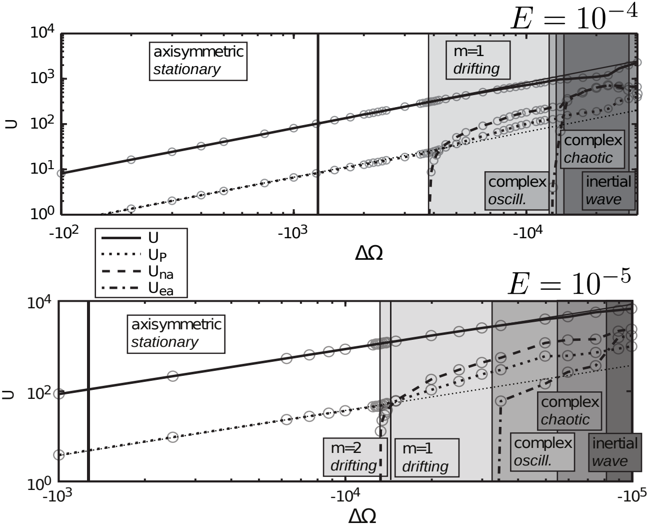
Zimmerman (2010)

Inertial Modes



(Wicht, 2014)

Inertial Modes



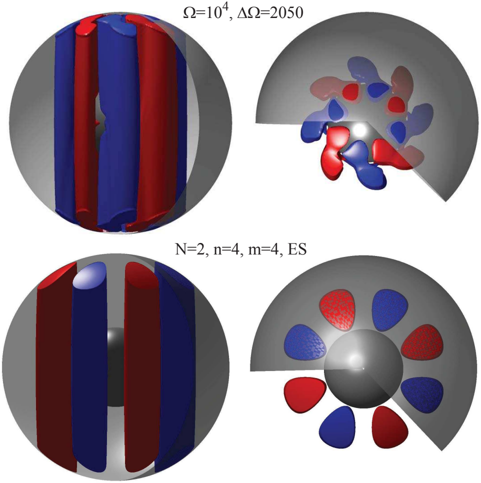
(Wicht, 2014)

Inertial Modes

- Previously found in experiments (Kelley et al., 2007) and in simulations (Matsui et al., 2011)
- They are solutions of linearized inviscid Navier-Stokes equation (Zhang et al., 2001):

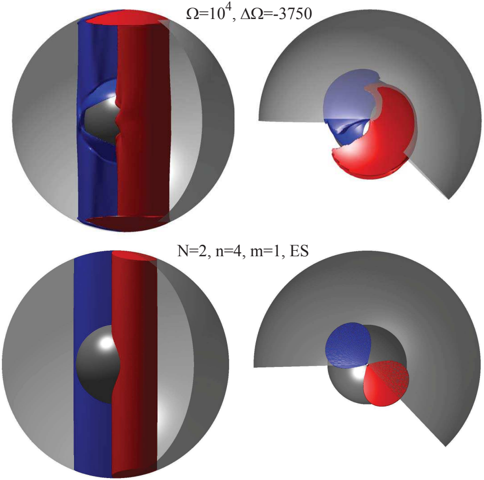
$$\frac{\partial \mathbf{U}}{\partial t} = -\nabla p - 2\Omega \hat{\mathbf{z}} \times \mathbf{U}$$

Inertial Modes



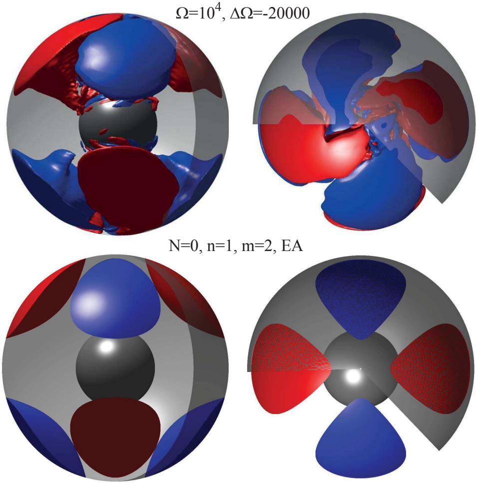
(Wicht, 2014)

Inertial Modes



(Wicht, 2014)

Inertial Modes



(Wicht, 2014)

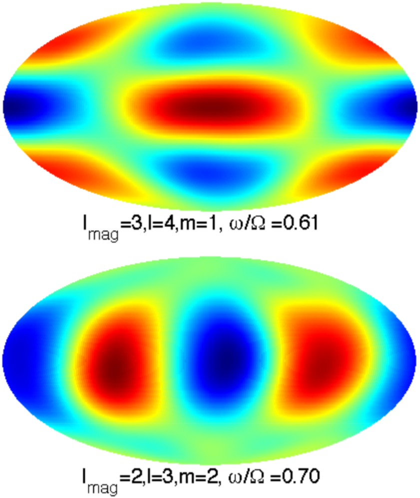
Inertial Modes

Value of frequency $(\omega/2\Omega)$

Mode	Our simulations	Analytical
m = 2	0.36	0.333
m = 1	0.3060	0.31

Mode	Rieutord et al. (2012)	(Kelley et al., 2007)
m = 2	0.3526	0.35
m = 1	0.31	0.305

Magnetic signature of inertial modes

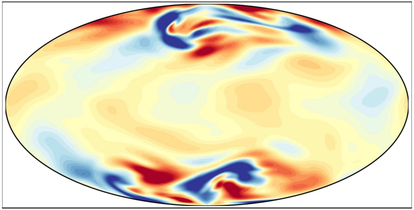


Kelley et al. (2007)

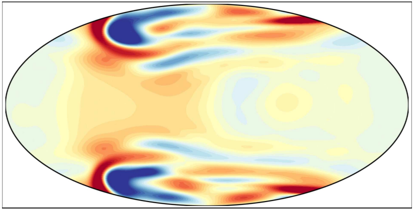
External magnetic field

We apply a uniform axial magnetic field

External magnetic field



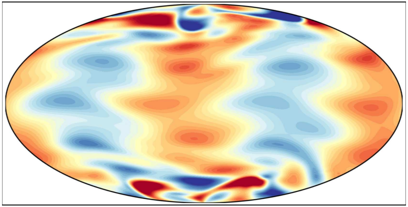
(a) Weak Field



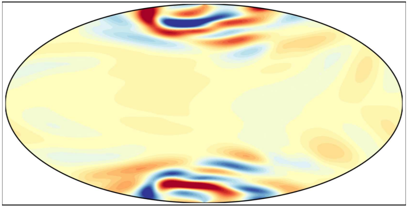
(b) Strong Field

Figure: Radial Velocity

External magnetic field



(a) Weak Field



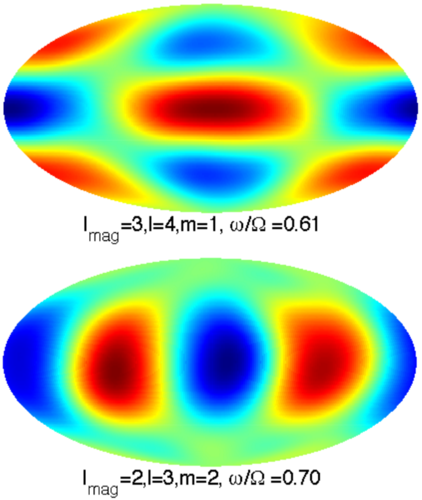
(b) Strong Field

Figure: Cylindrically radial magnetic field

External magnetic field

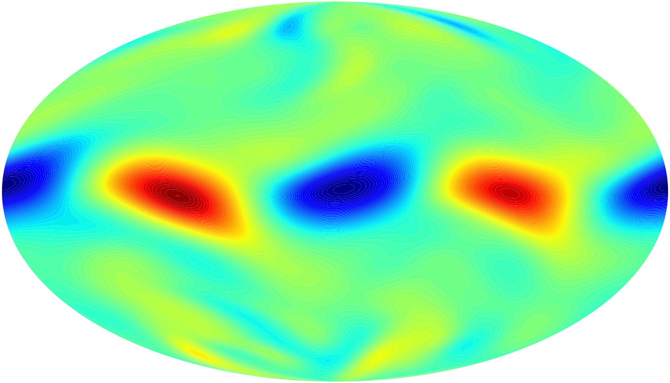
Not realistic for experiment! Use current loop instead!

Magnetic signature of inertial modes

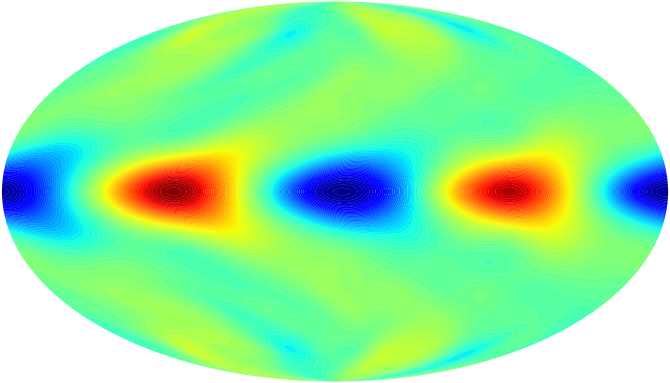


Kelley et al. (2007)

External current loop



Cheating a bit. . . (without success)



Conclusions and Outlook

- Simulations getting closer to experiment in terms of parameters, but still some way to go
- Simulations might be in a similar regime since we observe the same inertial modes
- Inertial modes DO NOT like a magnetic field with same symmetry
- Inertial modes get killed if magnetic field amplitude is increased in order to attain dynamo action
- To preserve inertial modes, a magnetic field of opposite symmetry must be imposed
- Next steps → to get magnetic signatures right and to simulate magnetic sensors

References

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