Observational constraints on the solar dynamo

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Convection theory predicts a high diffusivity – dynamo models require a low diffusivity.

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Figure 1. Different diffusivity profiles used in kinematic dynamo simulations. The solid black line corresponds to an estimate of turbulent diffusivity obtained by combining mixing-length theory (MLT) and the Solar Model S. The fact that viable solutions can be obtained with such a varied array of profiles has led to debates regarding which profile is more appropriate. Nevertheless, it is well known that kinematic dynamo simulations cannot yield viable solutions using the MLT estimate.

What we can observe is the butterfly diagram.
Sunspots are a measure of toroidal flux
What we measure:
Sunspot Number \( (t) \)
Mean \( |\hat{\lambda}|(t) \)
HWHM\((t)\)

(t based on years)

What we can observe is the butterfly diagram.
Waldmeier effect

--- Sunspot Number (t)

From Waldmeier 1941
Waldmeier effect

\[ \text{Mean } |\lambda|(t) \]

\[ \text{-- Sunspot Number}(t) \]

From Waldmeier 1955
Waldmeier effect

Sunspot Number(t) --

-- Mean $|\lambda|(t)$

From Waldmeier 1955
Waldmeier effect

Sunspot Number(t) vs Mean $|\lambda|(t)$

From Waldmeier 1955
We can plot measured quantities against one another.

Sunspot Number \((t)\)
Mean \(|\lambda| (t)\)
HWHM (t)

(t based on years)
HWHM(t) vs Sunspot Number (t)

Different colors represent different cycles

See also: Ivanov et al 2011 & Miletsky et al 2011
And cycle-average studies: Solanki, Wenzler & Schmitt, 2008,
Jiang, Cameron, Schmitt & Schüssler (2011) & : Hathawy 2015
Waldmeier effect

Sunspot Number(t)
vs
Mean $|\lambda|(t)$

HWHM(t) vs Mean $|\lambda|(t)$
Waldmeier effect

Sunspot Number(t) vs Mean $|\lambda|(t)$

Sunspot number decreases when centre of active region belt moves to within 1.8 HWHM of the equator!
Interpretation

Weak cycle

Strong cycle
Interpretation

Weak cycle

Strong cycle
Determining the turbulent diffusivity

Model with
\[
\frac{R_\odot^2}{R_c^2} \eta_{\text{turb}} = 600 \text{ km}^2\text{s}^{-1}.
\]

\[
\Rightarrow \quad 250 \text{ km}^2\text{s}^{-1} \leq \eta_{\text{turb}} \leq 600 \text{ km}^2\text{s}^{-1}.
\]
Different diffusivity profiles used in kinematic dynamo simulations. The solid black line corresponds to an estimate of turbulent diffusivity obtained by combining mixing-length theory (MLT) and the Solar Model S. The fact that viable solutions can be obtained with such a varied array of profiles has led to debates regarding which profile is more appropriate. Nevertheless, it is well known that kinematic dynamo simulations cannot yield viable solutions using the MLT estimate.
Conclusions: I

Turbulent diffusivity is 250—600 km$^2$/s where the toroidal flux is located.
Evolution of the Sun’s toroidal flux II
The solid black line corresponds to an estimate of turbulent diffusivity obtained by combining mixing-length theory (MLT) and the Solar Model S. The fact that viable solutions can be obtained with such a varied array of profiles has led to debates regarding which profile is more appropriate. Nevertheless, it is well known that kinematic dynamo simulations cannot yield viable solutions using the MLT estimate.
The butterfly wings maintain a HWHM of 6 degrees for many years (weak cycles) or the HWHM decreases (strong cycles). Why don‘t the butterfly wings diffuse outwards?

\[ \eta_{\text{turb}} = 250 \text{ km}^2/\text{s} = 53 (\degree)^2/\text{year} \]

\[ \approx 7^2 (\degree)^2/\text{year} \]

Suggests \( \eta_{\text{turb}} \ll 250 \text{ km}^2/\text{s} \)?
What we can observe is the butterfly diagram.
The average butterfly wings

![Graph showing the average butterfly wings over a range of years.](image)
The flow required to prevent diffusion

\[ \nabla \times (U \times B) = \nabla \times \eta_{\text{turb}} \nabla \times B. \]

Terms relevant for latitudinal expansion:

- \( U_\theta B_\phi \), Adveective
- \( U_\phi B_\theta \), Source
- \( \nabla \times \eta_{\text{turb}} \nabla \times B \), Diffusive

Source is distributed, and is locally substantially smaller than the other two terms (in the Sun).
The flow required to prevent diffusion

\[-\frac{1}{r} \frac{\partial U_\theta B_\phi}{\partial \theta} = \frac{\eta}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial B_\phi}{\partial \theta} \right) - \frac{\eta B_\phi}{r^2 \sin^2(\theta)} \]

\[-\frac{1}{r} \frac{\partial U_\theta B_\phi}{\partial \theta} = \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\eta}{r \sin(\theta)} \frac{\partial \sin(\theta) B_\phi}{\partial \theta} \right) \]

\[U_\theta B_\phi = \frac{\eta_{\text{turb}}}{r \sin(\theta)} \frac{\partial \sin(\theta) B_\phi}{\partial \theta} . \]

\[U_\theta = \frac{\eta_{\text{turb}}}{r \sin(\theta) B_\phi} \frac{\partial \sin(\theta) B_\phi}{\partial \theta} . \]
The flow required to counter diffusive expansion

\[ U_\theta = \frac{\eta_{\text{turb}}}{r \sin(\theta) B_\phi} \frac{\partial \sin(\theta) B_\phi}{\partial \theta}. \]
Flow required to counter diffusive expansion

Observed inflows at the surface, return flow at 50Mm(?)

Strong flows near equator to counter diffusion are not present in the observations – this is why we have strong cross-equator diffusion.
Conclusions: II

- Inflows into activity belt necessary to maintain butterfly wings.
- Butterfly diagram is essentially nonlinear.

- Toroidal field is strongly affected by the inflows.
The inflows are not axisymmetric
Questions.