Optimization of the Kinematic Dynamo

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Introduction

Many planets possess a magnetic field, which is created spontaneously in the interior via dynamo action. In which way is this process most efficient? To answer this question, we simulate dynamo action in a confined domain and optimize the initial conditions of a conductive flow field **U** and a magnetic field **B**. After a fixed time τ has passed, the optimal configurations give the maximal growth of the magnetic field [1].

Method

- Kinematic dynamo model with steady velocity field. First tested in a unit cube. Optimize dynamo action in an Earth-like sphere is being developed.
- Lagrange multiplier method for optimization. All variation of the Lagrangian with respect to independent variables $\delta\mathcal{L}/\delta x^i$ vanish at the optimum.
- Modified Newton's method for searching the optimal initial conditions. Random start to avoid being trapped at local minima.
- Pseudo-spectral method for solving a 2nd order PDE. Predictor-corrector method for time stepping.

Optimal configurations

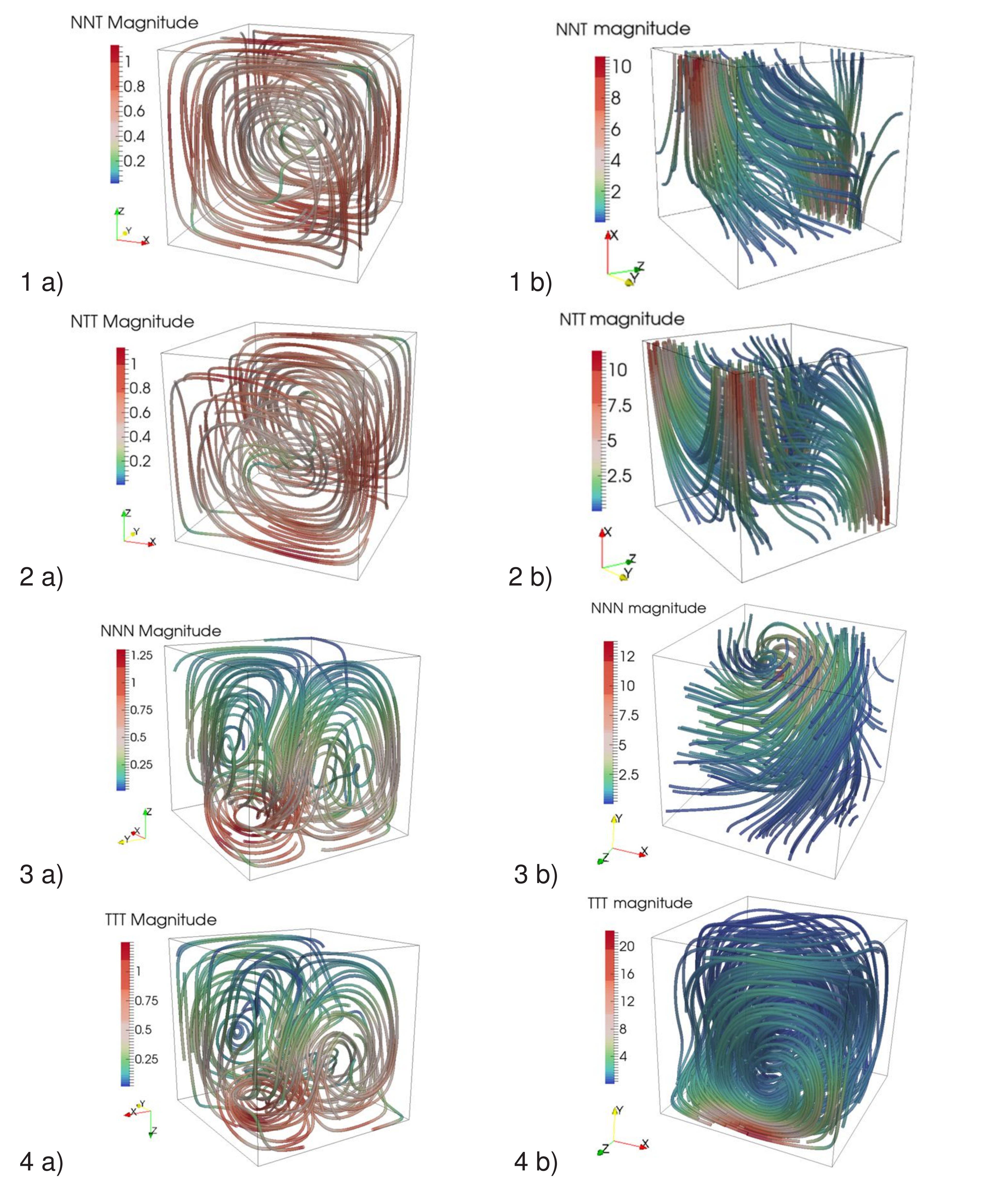

Fig. 1: 1=NNT, 2=NTT, 3=NNN, 4=TTT. a) stremlines of **U**, b) streamlines of **B** after time τ.

Equations

The Lagrangian where $\langle\cdots\rangle = \frac{1}{v}\int\cdots\, dv$:

$$\mathcal{L} = \ln\left\langle \mathbf{B}_\tau{}^2\right\rangle - \lambda_1\left(\left\langle(\nabla\times\mathbf{U})^2\right\rangle - 1\right) - \lambda_2\left(\langle\mathbf{B}_0^2\rangle - 1\right) - \langle\Pi_1\nabla\cdot\mathbf{U}\rangle - \langle\Pi_2\nabla\cdot\mathbf{B}_0\rangle - \int_0^{\mathcal{T}}\left\langle \mathbf{B}^\dagger\cdot\left[\partial_t\mathbf{B} - \nabla\times(\mathbf{U}\times\mathbf{B}) + \frac{1}{R_m}\nabla^2\mathbf{B}\right]\right\rangle dt \quad (1)$$

Update of **U** and $\mathbf{B}_0$ in spectral space for each mode **m** with Δ_1 and Δ_2 adjustable:

$$\widehat{\mathbf{U}}(\mathbf{m}) := \widehat{\mathbf{U}}(\mathbf{m}) + \frac{\Delta_1}{2\lambda_1\pi^2 m^2}\widehat{\frac{\delta\mathcal{L}}{\delta\mathbf{U}}}(\mathbf{m}) \quad (2)$$

$$\widehat{\mathbf{B}}_0(\mathbf{m}) := \widehat{\mathbf{B}}_0(\mathbf{m}) + \frac{\Delta_2}{2\lambda_2}\widehat{\frac{\delta\mathcal{L}}{\delta\mathbf{B}_0}}(\mathbf{m}) \quad (3)$$

Convergence criteria:

$$\sqrt{\left\langle\frac{\delta\mathcal{L}}{\delta\mathbf{u}}^2\right\rangle + \left\langle\frac{\delta\mathcal{L}}{\delta\mathbf{B}}^2\right\rangle} < 10^{-3} \quad (4)$$

Critical R_m

We use impermeable boundary condition for **U** and four types of boundary conditions [3] for **B** shown in Figure 2.

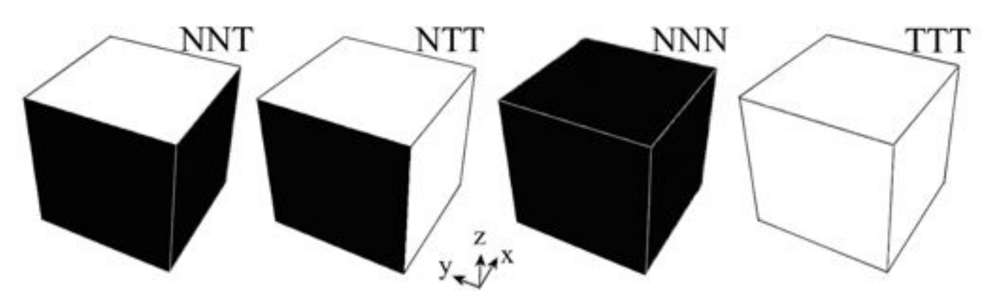

Fig. 2: N: normal/pseudo-vacuum. T: tangential/superconducting.

The magnetic Reynolds number R_m defined below is an indicator of the efficiency of the dynamo. At the critical R_m is when the growth rate of the magnetic energy γ defined below is zero.

$$R_m = \frac{\left\langle(\nabla\times\mathbf{U})^2\right\rangle^{1/2} L^2}{\eta}, \quad \gamma = 2\frac{d\ln\mathbf{B}}{dt} \quad (5)$$

For each simulation, γ is measured after a sufficiently large time window. The critical R_m is found through a linear interpolation in Figure 3. The main properties of the optimal configurations at critical R_m are listed in Table 1 and their visualization is shown in Figure 1

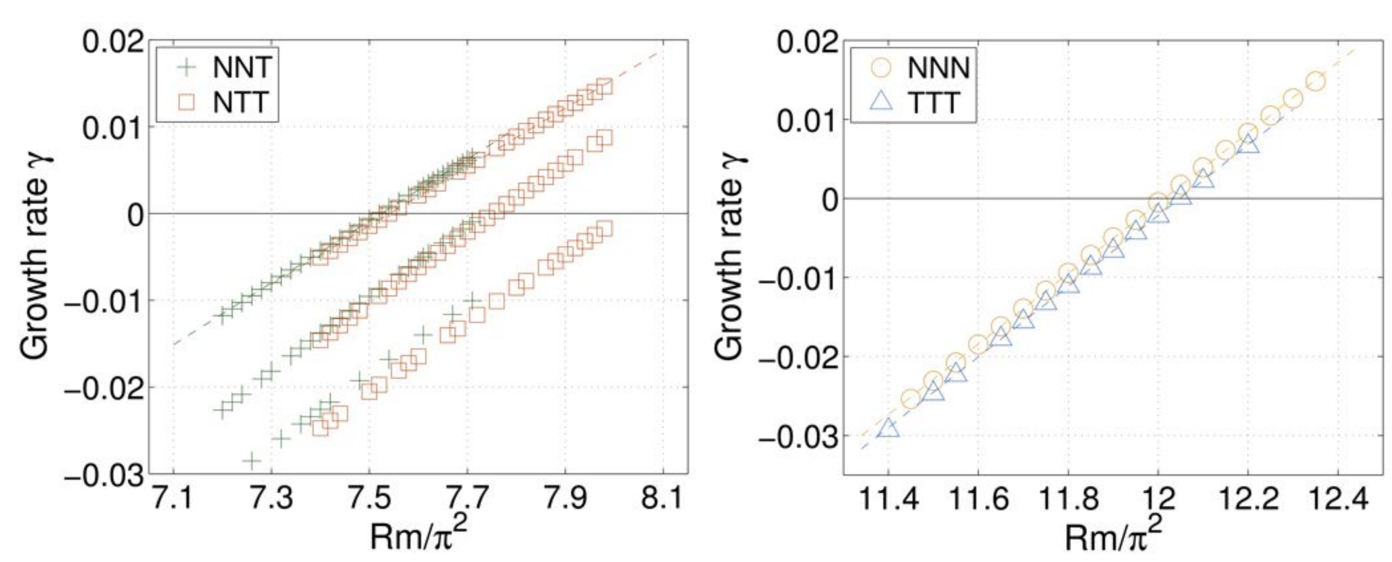

Fig. 3: Optimized growth rate γ versus R_m. Left: NNT & NTT pair. Right: NNN & TTT pair.

	NNT	NTT	NNN	TTT
Critial R_m/π^2	7.52	7.54	12.01	12.05
Mean helicity	0	0	0.18	0.21
RMS velocity	0.59	0.60	0.33	0.34

Table 1: Properties of the optimal flow.

Conclusion

In this project we have identified the most efficient dynamo in a unit cube using a Lagrange multiplier method. The critical R_m are found for four setups with different types of magnetic boundary conditions. Compared with known dynamos, the critical values are much lower. As for the optimal configurations, there exist pair-wise similarities. The optimal flow field has zero helicity for the NNT and NTT pair, but non-zero values for the other pair. The root mean square velocity in all setups ranges from 0.33 to 0.60. It is observed that swapping the magnetic boundary from T to N leaves the magnetic energy growth rate nearly unchanged, for which we can relate to [2]. We are developing a model to optimize the kinematic dynamo in a sphere. Since the flow field depends on the geometry, we expect a different configuration at the optimum.

References

[1] Willis, Ashley P. *Optimization of the Magnetic Dynamo.* Physical review letters 109.25 (2012): 251101.

[2] Favier, B. and M. R. E. Proctor. *Growth rate degeneracies in kinematic dynamos.* Physical Review E 88.3 (2013): 031001.

[3] Krstulovic, Giorgio, et al. *Axial dipolar dynamo action in the Taylor-Green vortex.* Physical Review E 84.6 (2011): 066318.