Scaling Laws in Rotating Convection

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Motivation

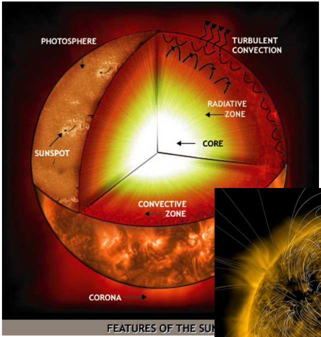
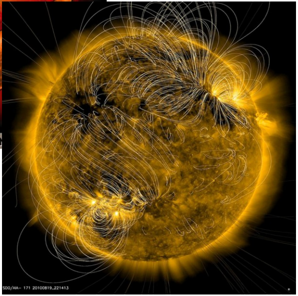


Image: NASA



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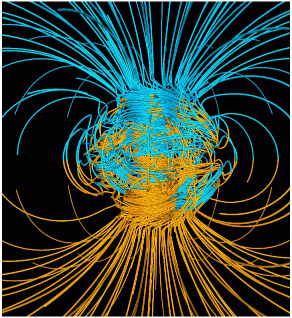


Image: G. Glatzmaier, UCSC

Introduction - scaling laws

- Seek scaling laws between e.g., heat flux, magnetic field, flow velocity and other characteristics e.g., rotation rate.

$$Nu \propto Ra^{\beta}$$

β depends on the regime and we are yet to develop a comprehensive theory to explain it.

- To simplify studies, local box simulations are often used in place of more complicated geometries (spheres).

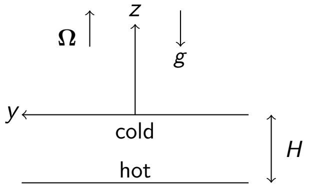
- Many studies including some theoretical e.g., Grossman & Lohse (2000), some experimental e.g., Ahlers et al. (2009) and some numerical e.g., Schmitz & Tilgner (2009).

Introduction - literature

- Barker et al. (2014) wanted to test predictions made by a rotating mixing length theory (MLT) using numerical simulations.
  - Non-rotating MLT predicts $\beta = \frac{1}{2}$ (Kraichnan (1962)) - yet to be found in simulations where $\beta = \frac{2}{7}, \frac{1}{3}$.
  - Malkus (1954) considered the behaviour of the boundary layers and obtained $\beta = \frac{1}{3}$.
- King et al. (2012), extended the theory of Malkus to incorporate rotation $\rightarrow \beta = 3$ in the geostrophic regime.
- Cheng et al. (2015) found $\beta \uparrow$ as $Ek \downarrow$ with $\beta > 3$ at the smallest Ekman numbers.
- Barker et al. (2014) focussed on the bulk properties and tested the rotating MLT first proposed by Stevenson (1979). $\rightarrow$ Focus on this approach today.

Outline of talk

A simple model



$\mathbf{\Omega} = \Omega(0, 0, 1)$

Boussinesq fluid

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\nabla p + T\hat{\mathbf{e}}_z + \nu \nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T$$

- T is a scaled temperature (cf. buoyancy).

Mixing Length Arguments

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- Consider the rapidly rotating limit, i.e., $2\Omega \gg N_*$
$\Rightarrow$ Modes with $k_\perp > k_z \frac{2\Omega}{N_*}$ have growth rate $\sigma \approx N_*$.

Mixing Length Arguments II

- Assume modes with largest wavelengths are responsible for the bulk of the heat transport (Malkus (1954))
  $\rightarrow$ The largest wavelength in a convection zone of depth H is

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- Combining the above relations and assuming $u_\perp \sim w$ gives ...

Numerical simulations

$$\ldots \boxed{-\frac{d\langle T\rangle}{dz} = N_*^2 \sim \frac{F^{\frac{2}{5}}\Omega^{\frac{4}{5}}}{H^{\frac{4}{5}}}} \quad \boxed{w \sim \frac{H^{\frac{1}{5}}F^{\frac{4}{5}}}{\Omega^{\frac{1}{5}}}} \quad \boxed{T' \sim \frac{F^{\frac{3}{5}}\Omega^{\frac{1}{5}}}{H^{\frac{1}{5}}}}$$

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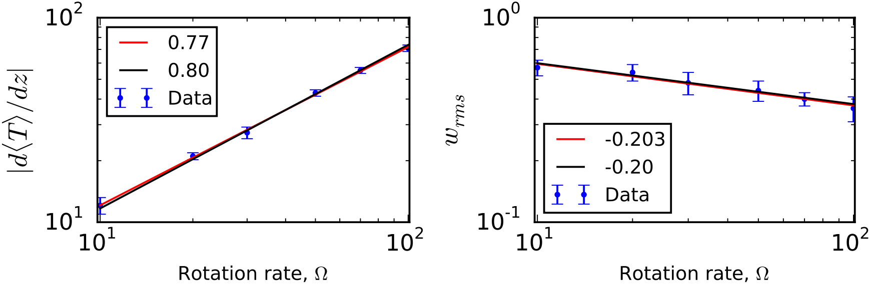
- Fix

$$F = -\kappa\frac{d\langle T\rangle}{dz} + \langle wT\rangle = 1,$$
$$H = 1.$$

- This reduces the scaling laws to be dependent on Ω only.
- Solve equations numerically using a pseudospectral code: 'Dedalus' (www.dedalus-project.org) with inputs ν, κ, Ω and stress free boundary conditions.
- To assess bulk behaviour, evaluate desired quantities ($\frac{d\langle T\rangle}{dz}$ etc) at mid-depth.

Numerical results: vertical rotation



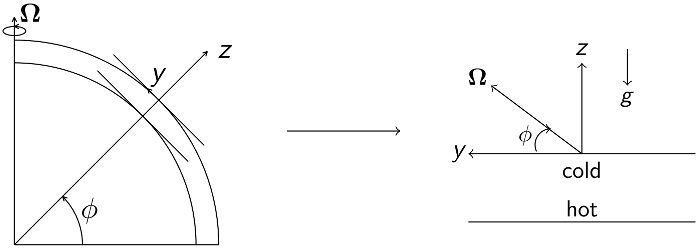
- Independent of diffusivities.
- Barker et al. (2014) used a heating/cooling setup to avoid boundary layers and obtained similar results.
- Provides support for reduced equations of Julien et al. (2012) (cf. Stellmach et al. (2014)).

Extensions to the model

- More realistic geometry:

  Plane layer $\rightarrow$ Tilted plane layer $\rightarrow$ Spherical shells?

- As an intermediate step consider the tilted plane layer where gravity and rotation are oblique.
- Rotation vector is given by $\mathbf{\Omega} = (0, \Omega\cos\phi, \Omega\sin\phi)$.



- Stevenson's (1979) theory proposes $\Omega \rightarrow \Omega\sin\phi$.

Tilted rotation - preliminary results

$$-\frac{d\langle T\rangle}{dz} = N_*^2 \sim \frac{F^{\frac{2}{5}}\,\Omega^{\frac{4}{5}}\,(\sin\phi)^{\frac{4}{5}}}{H^{\frac{4}{5}}}$$

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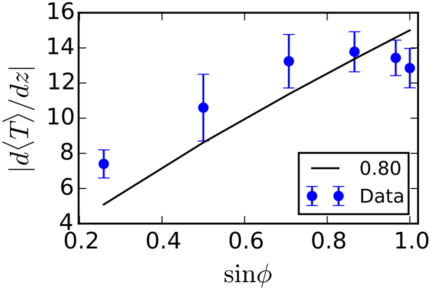

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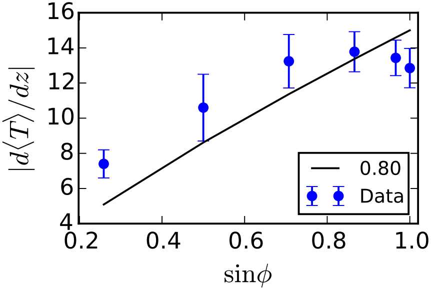

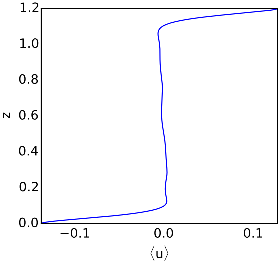

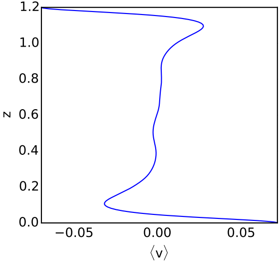

Issues:

Effect of horizontal box size on mean flow?

How to define H?

Effect of boundary layers?

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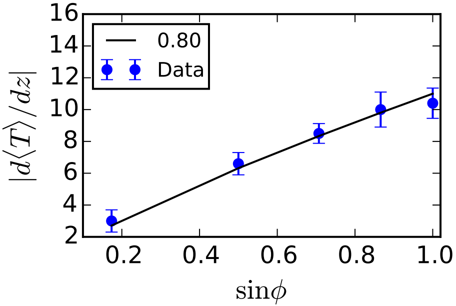

- Heating/cooling setup of Barker et al. (2014).
- No mean flows at these parameters.
- Still issue of horizontal box size.

Conclusions

Summary

- Presented the key physical arguments of a rotating MLT as first proposed by Stevenson.
- The theory predicts well the behaviour of the bulk convection in the rapidly rotating limit when rotation and gravity are aligned (cf. Barker et al. (2014)).
- Extended to the case where rotation and gravity are oblique, mean flows driven (especially in the boundary layers) appear to complicate things.

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Current and future work

- How do mean flows affect the heat transport? What is the role of the boundary layers?

- Include a background density gradient $\rightarrow$ Introduces an asymmetry between upflows and downflows but can temperature be replaced with entropy in the above theory?

- Inclusion of a magnetic field?