A parameter study of Jupiter-like dynamo models

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– ... and and recent Jupiter models (Gastine et al. 2014; Jones 2014): electrical conductivity, and density gradient
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Christensen et al. (2006, 2010) showed the importance of studying the control parameters systematically to identify the physical processes.
• We collected numerical models based on Jupiter’s interior density profile (Nettlemann et al., 2012) and polytropic models with similar number of density scale heights across the shell, both with simplified versions of Jupiter’s electrical conductivity profile (French et al., 2012)
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Upcoming missions to Jupiter will improve the observational models: Juno, launched in 2011 and planned to reach Jupiter in 2016.
**Model**

- Variable conductivity profiles based on French et al. 2012, approximated by a profile composed of 2 branches (Gómez-Pérez, 2010):
  - A polynomial to model the inner metallic layer
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• Density profiles considered:
  – Polytropic density profile of 5 density scale heights ($N_\rho=5$) and index $m=2$
Motivated by Christensen et al. (2010), we selected a set of parameters to compare with Jupiter observations:

- Parameters based on the first degrees of the magnetic spectra, due to limited available observational data for comparison.
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Parameters chosen:

- Axial dipolarity of the surface magnetic field: \( f_{\text{dip}}(\ell_{\text{max}} = 4) \)
- Tilt angle of the dipole: \( \theta_{\text{dip}} \)
- Ratio dipole/quadrupole: \( (\ell = 1)/(\ell = 2) \)
- Ratio dipole/octupole: \( (\ell = 1)/(\ell = 3) \)
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Following Christensen et al. (2010), an RMS error can be determined from the 4 parameters:

\[
\text{RMSerr} = \sqrt{(f_{dip} - f_{dipJ})^2 + (\theta_{dip} - \theta_{dipJ})^2 + ((\ell=1)/(\ell=2) - (\ell=1)/(\ell=2))_2^2 + ((\ell=1)/(\ell=3) - (\ell=1)/(\ell=3))_2^2} / 4
\]
Conditions for Jupiter-like models

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Choosing the control parameters

- Following Christensen et al. (2010), we defined a wedge that incorporates the numerical models and extrapolate to Jupiter’s parameters
  - Some of the best models appear to be near the upper boundary of the cluster of points
  - Both the best models and Jupiter’s point are not far from the central axis of the wedge
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But...

- Edges are poorly constrained.
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• But...
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• This criteria does not seem to work to distinguish best from worst
• Simplifying the electrical conductivity profile
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  – Models that best match appear to concentrate around 89 – 92% of the radius
Example of best models

- All models follow closely the VIP4 observational model, but numerical models tend to have a stronger zig-zag pattern

Br: $r/\rho_o = 1.000$
- $E_\eta = 2.6 \times 10^{-5}$

Br: $r/\rho_o = 1.000$
- $E_\eta = 5.0 \times 10^{-5}$

$B_\eta$: $r/\rho_o = 1.000$
- $E_\eta = 3.9 \times 10^{-5}$

Br: $r/\rho_o = 1.000$
- $E_\eta = 4.1 \times 10^{-5}$
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Selecting conductivity profile

- Smaller conductivity decay already reproduces the spectral magnetic features below $\ell=5$
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- Most of the resolution covered by Juno can be modeled with decay of 3 orders of magnitude
- The best reproduction in numerical models is with 4 orders of decay in the outer layer
• Two orders of magnitude decay:
  – there is little difference in the magnetic spectra at $E=1\times10^{-4}$ and curiously all show that the zig-zag pattern breaks at $\ell = 5$
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Gastine et al. (2014) conductivity profile at $E=1 \times 10^{-4}$:
  - There appears to be no difference in the field morphology between the two models
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  – Little difference, but a slight increase for the Jupiter models at the intermediate scales, in the range $\ell=6-14$
**Polytrope vs. Jupiter model**

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- **Gastine et al. (2014) conductivity profile at \( E=1\times10^{-5} \):**
  - Similar enhancement at scales below \( \ell=20 \), observed at the higher Ekman number
Linear+exponential (Jupiter-like) conductivity profile vs. constant+exponential conductivity profile

- With the first, it is difficult to reach $Rm=50$ above 90% of the radius for the values of Ekman number modeled.
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• The simplified profile of the best models has a maximum of $Rm$ near the outer boundary, which facilitates having a high enough value close enough to the surface
In this parameter study, we conclude that the differences between a realistic Jupiter interior density and a polytrope of 5 density scale heights are not significant, at first approximation.

The electrical conductivity profile may also at first glance be simplified by a profile that allows reaching $Rm=50$ around 89 – 92% of the outer radius.
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Future additions:
- Add other density contrasts to the parameter plots (mainly low density contrasts) to complete rule out the simple Boussinesq models as possibly “good enough”
- Add Saturn’s data, to compare with the numerical models, since some tend to be very axisymmetric
- Adding a parameter that incorporates comparison of the flow

Outlook
Thank you.
Thank you.
Force balance

no IH

IH

IH + Pr=0.1

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