Unified scaling relation for rising flux tubes in solar-like stars

Y. Fournier, R. Arlt, U. Ziegler and K. G. Strassmeier

Leibniz Institute for Astrophysics
# Overview

## Motivations

## Numerical experiments
- Numerical Setup
- Numerical Tool

## Results
- Scaling behavior
- Extension to 3D
- A universal relation for the rising time

## Application to BL-Dynamo

## Discussion and Conclusion
Motivations

Studying the dynamics of magnetic flux tube in solar-like interiors

- To constrain the BL-Dynamo
- To explain formation of active regions
- To explain active latitudes and title angles

Figure: Final stage of the rise of an emerging magnetic flux tube in a solar-like interior.
**Compressible simulations: the setup**

### Features
- Fully compressible MHD equations
- Spherical geometry in 2D and 3D
- Rotation

### Initial Conditions
- Entropic perturbation of various azimuthal wavenumbers (called $m$)

---

Figure: Description of the initial condition of the setup (3D case). Fournier et al. 2015 (in prep.)
Compressible simulations: *the tools*

The NIRVANA code.

- Spherical geometry
- Parallelized (good scaling behavior)
- Adaptive Mesh Refinement

Figure: Non homogeneous block structure distribution around the magnetic flux tube with 2-levels of refinement. Each block contains 4 cells (in 2D).
Compressible simulations: the standard simulation

Two forces are mainly acting on the flux tube:

1. **Buoyant force**: controlled by \( \beta = P/P_m \)
   
2. **Coriolis force**: controlled by \( M_{\text{rot.}} = \varpi \Omega_0/c_s \)

Figure: Magnetic contours of the flux tube at different times, showing the path taken by the flux tube while rising.
Compressible simulations: *the difficulties*

Compressibility → resolving sound waves.

- **Advantages:** general case, permits strong entropic deviations, enables helioseismologic studies, this is the *first specific* study.
- **Disadvantages:**
  - much longer simulations (short time step to catch fast sound waves)
  - impossibility to use a background reference state
  - limited in resolving the entropic signature of the flux tube

\[ \beta \text{ parameter is much too low} \]

solar \( \beta = 10^5 \gg \) simulation \( \beta = 100 \)

three orders of magnitude \( \neq \)!!

How can we be sure that the simulations are in the solar regime? Does the simulation scale?
Validation of the setup: \textit{2D experiment}

If a strongly buoyant flux tube in a fast rotating star shows the same characteristics as a weakly buoyant flux tube in a slowly rotating star, then our setup scales.

As shown by Schüssler and Solanki (1992) what ever pair of \((\beta, M_{\text{rot.}})\) which gives a constant \(Ro^*_m\) the results will be the same. \(Ro^*_m\) controls the regime of the rise.

\[ Ro^*_m = \frac{v_A}{\omega \Omega_0} \propto Ro_m \]

Hence, if \(M_{\text{rot.}}\) is chosen accordingly; a simulation with low \(\beta\) is in the correct regime.
Extension to 3D

- Stable
- Unstable

B > 0
B < 0

0.0 Prot. 0.4 Prot. 0.8 Prot. 1.2 Prot. 1.6 Prot.

Time

Scaling relation for rising flux tubes
Extension to 3D

Fan (2008) has shown that 2D and 3D rises of magnetic flux tubes are very different.

- 2D (axisymmetric): the conservation of angular momentum implies a deceleration which results in a latitudinal deflection. → high latitude of emergence.
- 3D (non-axisymmetric): The additional degree of freedom allows a redistribution of angular momentum. → low latitude of emergence, and tilt angles.

Questions

- What about the scaling behavior of the 3D rise?
- Does the scaling parameter hold?
Parameter study - 38 simulations

Figure: Each point represents a simulation in the $\left( \beta, M_{\text{rot}} \right)$ parameter space. It is not the best plot ever but still better than a long boring list.
Parameter study - 38 simulations

Figure: Each point represents a simulation in the \( (\beta, M_{\text{rot}}) \) parameter space. It is not the best plot ever but still better than a long boring list.
Parameter study - 38 simulations

Figure: Each point represents a simulation in the \((\beta, M_{\text{rot}})\) parameter space. It is not the best plot ever but still better than a long boring list.

Parameter space

- The darker points are simulations with a constant \(R_{\text{om}}^*\).
Parameter study - 38 simulations

Figure: Each point represents a simulation in the $(\beta, M_{\text{rot}})$ parameter space. It is not the best plot ever but still better than a long boring list.

Parameter space

- The darker points are simulations with a constant $R_{\text{om}}^*$. 

---

Y. Fournier
Scaling relation for rising flux tubes
Parameter study - 38 simulations

Figure: Each point represents a simulation in the $(\beta, M_{\text{rot}})$ parameter space. It is not the best plot ever but still better than a long boring list.

Parameter space

- The darker points are simulations with a constant $R_{O_m}^*$.
- No scaling for a constant $R_{O_m}^*$; the properties of the flux do not scale.
Parameter study - 38 simulations

Parameter space

- The darker points are simulations with a constant $R_{om}^*$.
- No scaling for a constant $R_{om}^*$: the properties of the flux do not scale.
- Does the setup scale at all?

Figure: Each point represents a simulation in the $(\beta, M_{rot.})$ parameter space. It is not the best plot ever but still better than a long boring list.
Looking for a scaling. \((\tilde{\tau}_{\text{rise}}, \beta, M_{\text{rot.}})\)
Looking for a scaling. \((\tilde{\tau}_{\text{rise}}, \beta, M_{\text{rot.}})\)
Looking for a scaling. \((\tilde{\tau}_{\text{rise}}, \beta, M_{\text{rot.}})\)
Looking for a scaling. \((\tau_{\text{rise}}, \beta, M_{\text{rot.}})\)
Looking for a scaling. \((\tau_{\text{rise}}, \beta, M_{\text{rot.}})\)
Reduction to 1D-function

The setup scales

- As a result of the scaling behavior of the setup we can reduce the relation to a 1D-function.

\[
\log(r_{\text{rise}}) = -\log\left(\frac{\beta^{2/5}}{M_{\text{rot.}}}\right)
\]
Reduction to 1D-function

As a result of the scaling behavior of the setup we can reduce the relation to a 1D-function.

- **one independent variable:**

  \[
  \left( \frac{1}{\beta} \right)^{2/5} \frac{1}{M_{\text{rot.}}}
  \]
What is the difference between 2D and 3D?

- 2D: $m=0$ → axisymmetric rise.
- 3D: $m>0$ → non-axisymmetric rise.

3 series varying azimuthal wavenumber ($m$):

- $m=0$:
  \[ Ro^\star = 0.92 \]  
  \[ \omega = 0.08 \]

- $m=4$:
  \[ (v_A)^0 = 0.85 \]  
  \[ (c_s)^0 = 0.15 \]  
  \[ \omega = \Omega_0 \]

- $m=8$:
  \[ (v_A)^0 = 0.85 \]  
  \[ (c_s)^0 = 0.15 \]  
  \[ \omega = \Omega_0 \]
What is the difference between 2D and 3D?

- Not the geometry, but the azimuthal wavenumber (m) of the perturbation.
  - 2D: m=0 → axisymmetric rise.
  - 3D: m>0 → non-axisymmetric rise.
What is the difference between 2D and 3D?

- **Not the geometry, but the azimuthal wavenumber (m) of the perturbation.**
  - **2D:** \( m=0 \) → axisymmetric rise.
  - **3D:** \( m>0 \) → non-axisymmetric rise.

- **3 series varying azimuthal wavenumber (m)**

<table>
<thead>
<tr>
<th>Series</th>
<th>Scaling parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m=0 )</td>
<td>( R_{0m}^* )</td>
</tr>
<tr>
<td>( m=4 )</td>
<td>( \frac{(v_A)^{0.92}(c_s)^{0.08}}{\varpi \Omega_0} )</td>
</tr>
<tr>
<td>( m=8 )</td>
<td>( \frac{(v_A)^{0.85}(c_s)^{0.15}}{\varpi \Omega_0} )</td>
</tr>
</tbody>
</table>
Defining a *unified* scaling parameter.

- In 3D the properties of the rise of the magnetic flux tube do not scale with the magnetic Rossby numbers, so we had to define a more general scaling parameter. We call it $\Gamma_\alpha$:

$$\Gamma_\alpha = \frac{(v_A)^\alpha (c_s)^{1-\alpha}}{\omega \Omega_0}$$

- Where $\alpha$ depends on the azimuthal wavenumber ($m$) of the perturbation.
Defining a *unified* scaling parameter.

▶ In 3D the properties of the rise of the magnetic flux tube do not scale with the magnetic Rossby numbers, so we had to define a more general scaling parameter. We call it $\Gamma_\alpha$:

$$\Gamma_\alpha = \frac{(v_A)^\alpha (c_s)^{1-\alpha}}{\varpi \Omega_0}$$

▶ Where $\alpha$ depends on the azimuthal wavenumber ($m$) of the perturbation.

▶ In the case of $m=0 \rightarrow \alpha=1$:

$$\Gamma_1 = \frac{v_A}{\varpi \Omega_0} = Ro_m^*$$
There is no difference between 2D and 3D.

- Not the geometry, but the azimuthal wavenumber (m) of the perturbation.
  - 2D: m=0 → axisymmetric rise.
  - 3D: m>0 → non-axisymmetric rise.

<table>
<thead>
<tr>
<th>Three series</th>
<th>One scaling</th>
<th>Three $\alpha$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>m = 0</td>
<td>$\Gamma_\alpha = \left(\nu_A\right)^\alpha \left(c_s\right)^{1-\alpha} \frac{1}{\varpi \Omega_0}$</td>
<td>1</td>
</tr>
<tr>
<td>m = 4</td>
<td></td>
<td>0.92</td>
</tr>
<tr>
<td>m = 8</td>
<td></td>
<td>0.85</td>
</tr>
</tbody>
</table>

- $\alpha$ is a function of the azimuthal wavenumber of the perturbation.
What do we learn about such a relation?

- The regime of the rise is driven by $\Gamma_\alpha$.
- We learn that it is possible to compare our simulations with stellar objects.
  - Even though the *numerical parameter space* is far away from the realistic one, $\Gamma_\alpha$ drives the regime, hence a simulation with the solar $\Gamma_\alpha$ can be compared with the sun.
What do we learn about such a relation?

- The regime of the rise is driven by $\Gamma_\alpha$.

- We learn that it is possible to compare our simulations with stellar objects.
  - Even though the numerical parameter space is far away from the realistic one, $\Gamma_\alpha$ drives the regime, hence a simulation with the solar $\Gamma_\alpha$ can be compared with the sun.

- And what about the dynamo?
Application: the Babcock-Leighton dynamo

- The BL-Dynamo needs a mechanism to transport magnetic flux from the bottom of the CZ to the surface. The traditional mechanism is magnetic buoyancy under the form of flux tubes.
- Usually people consider the rise to be instantaneous. Is it really?
- From our simulations we can constrain the rise.

Figure: A sketch of the Babcock-Leighton dynamo mechanism. Picture from Charbonneau (2010).
Evolution of the relative rising time with $\Gamma_\alpha$

- For an azimuthal wavenumber $m = 8$

$$\tilde{\tau}_{\text{rise}} \propto (\Gamma_{0.85})^{-1.17}$$
Future work: implementing a delayed BL-dynamo (testing phase)

- In a similar manner than Jouve et al. (2010) we are introduced a delay in the $\alpha$-effect.
- From our results we can constrain the delay (for $m=8$):

$$\tau_B = \left( \frac{\tau_0}{\tau_{\text{diff}}} \right) \sin(\theta) (\mathcal{M}_{\text{rot.}})^{0.17} \left( \frac{AB\phi}{B_{\text{Eq}}} \right)^{-1}$$

- We want to study the possible limit of the BL-Dynamo. Fast rotating stars has:
  - shorter cycles
  - longer rising time
Future work: implementing a delayed BL-dynamo (testing phase)

- In a similar manner than Jouve et al. (2010) we are introduced a delay in the $\alpha$-effect.
- From our results we can constrain the delay (for $m=8$):

\[
\tau_B = \left( \frac{\tau_0}{\tau_{\text{diff}}} \right) \sin(\theta) \left( M_{\text{rot.}} \right)^{0.17} \left( \frac{A B \phi}{B_{\text{Eq}}} \right)^{-1}
\]

- We want to study the possible limit of the BL-Dynamo. Fast rotating stars has:
  - shorter cycles
  - longer rising time
- What would happen if the rising time becomes of the same order of magnitude than the cycle?
Discussion and Conclusion

- We conduct numerical experiments of compressible magnetic flux tubes, which were able to reproduce results found in the literature.
- In 3D, the setup scales in a different manner than in 2D.
- We designed a general scaling parameter unifying former and current results:
  \[
  \Gamma_\alpha = \frac{(v_A)^\alpha (c_s)^{1-\alpha}}{\varpi \Omega_0}
  \]
- The setup scales. Therefore, even though the numerical domain is different from the realistic one, the simulations are in the solar regime.
- We could compute the rising time scale needed to constrain the BL-dynamo.
- We include this rising time as a delay in the \(\alpha\) effect for a BL-dynamo.