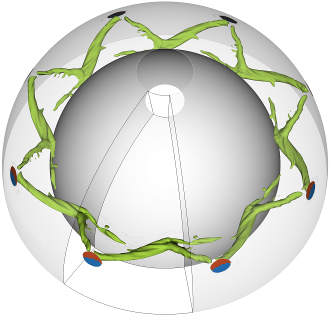
Unified scaling relation for rising flux tubes in solar-like stars



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Overview

Motivations

Numerical experiments

- Numerical Setup
- Numerical Tool

Results

- Scaling behavior
- Extension to 3D
- A universal relation for the rising time

Application to BL-Dynamo

Discussion and Conclusion

Motivations

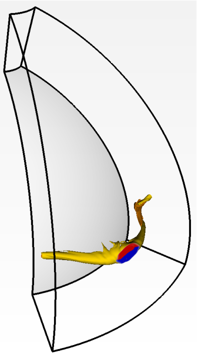


Figure : Final stage of the rise of an emerging magnetic flux tube in a solar-like interior.

Studying the dynamics of magnetic flux tube in solar-like interiors

- To constrain the BL-Dynamo
- To explain formation of active regions
- To explain active latitudes and title angles

Compressible simulations: *the setup*

Features

- Fully compressible MHD equations
- Spherical geometry in 2D and 3D
- Rotation

Initial Conditions

- Entropic perturbation of various azimuthal wavenumbers (called m)

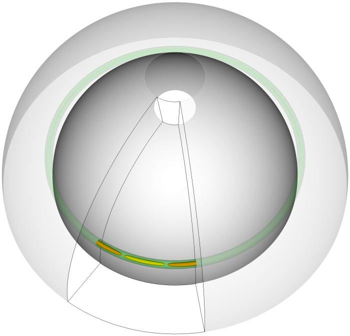


Figure : Description of the initial condition of the setup (3D case). Fournier et al. 2015 (in prep.)

Compressible simulations: *the tools*

The NIRVANA code.

- Spherical geometry
- Parallelized (good scaling behavior)
- Adaptive Mesh Refinement

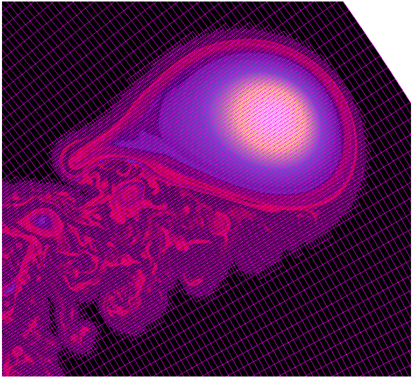


Figure : Non homogeneous block structure distribution around the magnetic flux tube with 2-levels of refinement. Each block contains 4 cells (in 2D).

Compressible simulations: *the standard simulation*

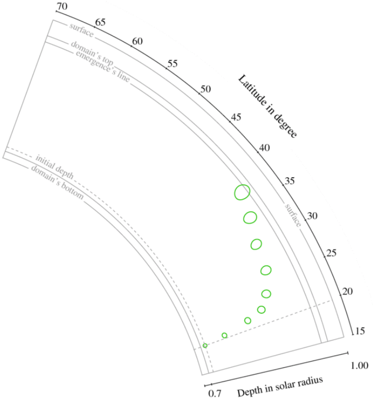


Figure : Magnetic contours of the flux tube at different times, showing the path taken by the flux tube while rising.

Two forces are mainly acting on the flux tube:

1. **Buoyant force**: controlled by $\beta = P/P_{\mathrm{m}}$
2. **Coriolis force**: controlled by $\mathcal{M}_{\mathrm{rot.}} = \varpi\Omega_0/cs$

Compressible simulations: *the difficulties*

Compressibility $\rightarrow$ resolving sound waves.

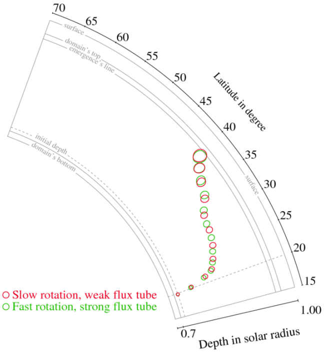
- Advantages: general case, permits strong entropic deviations, enables helioseismologic studies, this is the **first specific** study.
- Disadvantages:
  - much longer simulations (short time step to catch fast sound waves)
  - impossibility to use a background reference state
  - limited in resolving the entropic signature of the flux tube $\rightarrow \beta$ parameter is much too low

solar $\beta = 10^5 \gg$ simulation $\beta = 100$
three orders of magnitude $\neq$!!

How can we be sure that the simulations are in the solar regime?
Does the simulation scale?

Validation of the setup: *2D experiment*

If a strongly buoyant flux tube in a fast rotating star shows the same characteristics as a weakly buoyant flux tube in a slowly rotating star, then our setup scales.

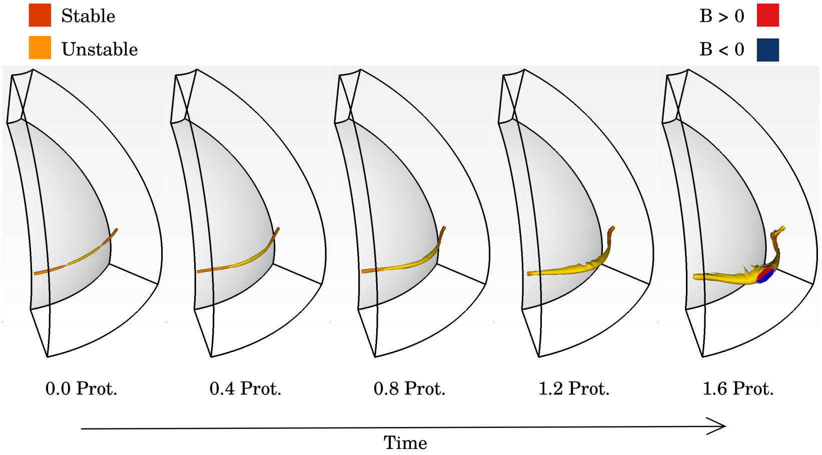


As shown by Schüssler and Solanki (1992) what ever pair of $(\beta, \mathcal{M}_{\mathrm{rot.}})$ which gives a constant $Ro^{\star}_{\mathrm{m}}$ the results will be the same. $Ro^{\star}_{\mathrm{m}}$ controls the regime of the rise.

$$Ro^{\star}_{\mathrm{m}} = \frac{v_{\mathrm{A}}}{\varpi \Omega_0} \propto Ro_{\mathrm{m}}$$

Hence, if $\mathcal{M}_{\mathrm{rot.}}$ is chosen accordingly; a simulation with low β is in the correct regime.

Extension to 3D



Extension to 3D

Fan (2008) has shown that 2D and 3D rises of magnetic flux tubes are very different.

- 2D (axisymmetric): the conservation of angular momentum implies a deceleration which results in a latitudinal deflection.
  $\rightarrow$ high latitude of emergence.
- 3D (non-axisymmetric): The additional degree of freedom allows a redistribution of angular momentum.
  $\rightarrow$ low latitude of emergence, and tilt angles.

Questions

- What about the scaling behavior of the 3D rise?
- Does the scaling parameter hold?

Parameter study - 38 simulations

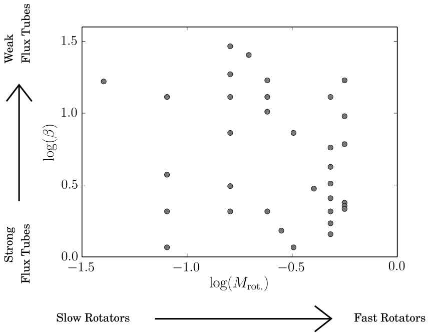
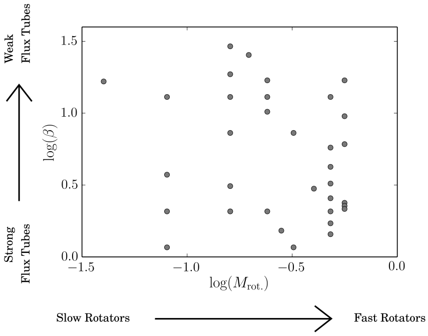


Figure : Each point represents a simulation in the (β, $\mathcal{M}_{\mathrm{rot.}}$) parameter space. It is not the best plot ever but still better than a long boring list.

Parameter space

Parameter study - 38 simulations



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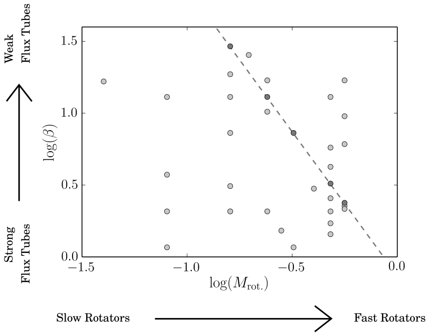

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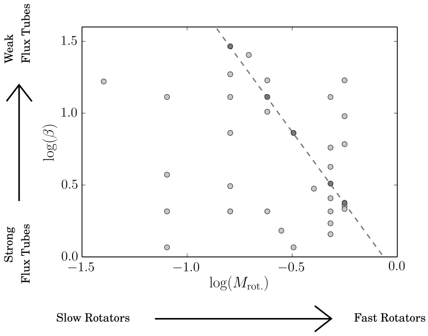

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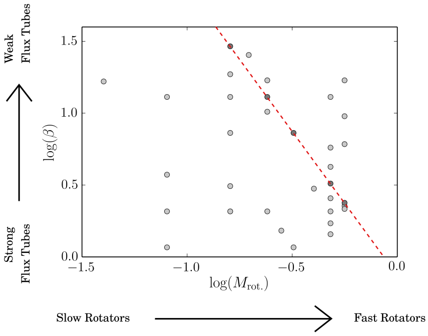

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Parameter space

- The darker points are simulations with a constant $Ro_{\mathrm{m}}^{\star}$.
- No scaling for a constant $Ro_{\mathrm{m}}^{\star}$; the properties of the flux do not scale.

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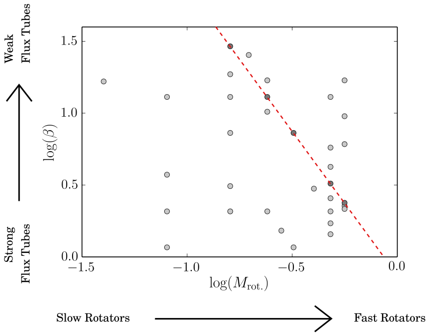

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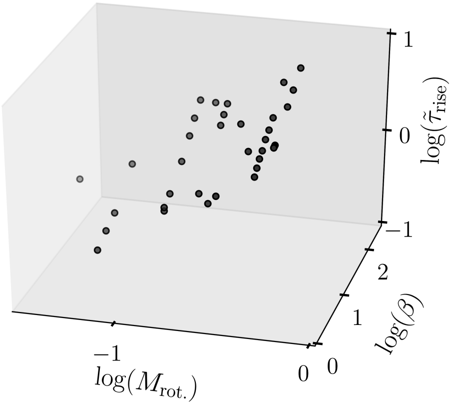
Parameter space

- The darker points are simulations with a constant $Ro^{\star}_{\mathrm{m}}$.
- No scaling for a constant $Ro^{\star}_{\mathrm{m}}$; the properties of the flux do not scale.
- Does the setup scale at all?

Looking for a scaling. $(\tilde{\tau}_{\text{rise}},\ \beta,\ \mathcal{M}_{\text{rot.}})$



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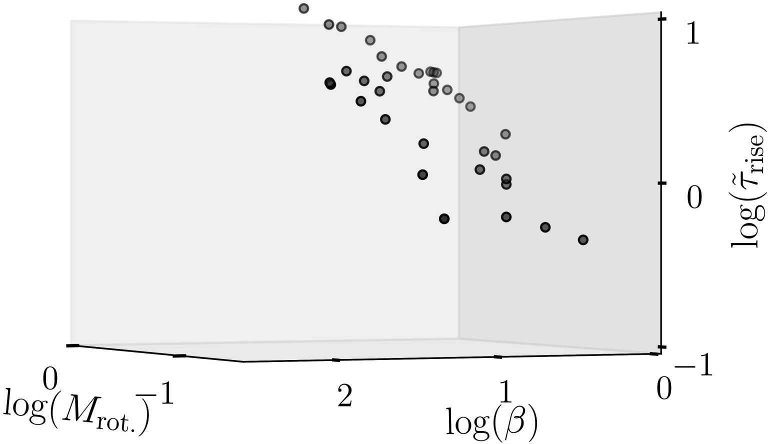

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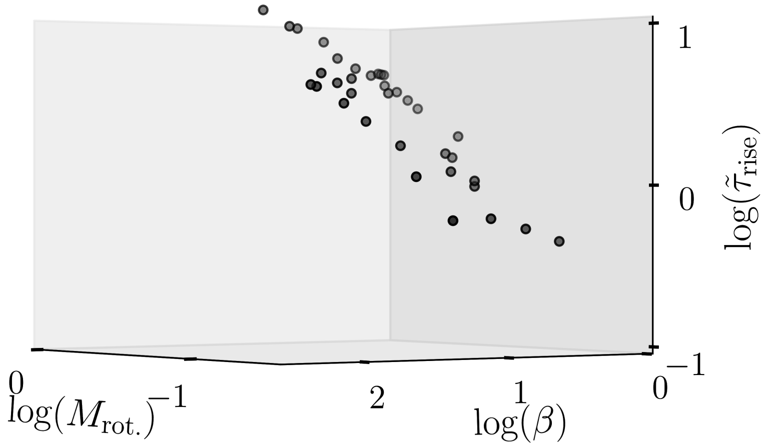

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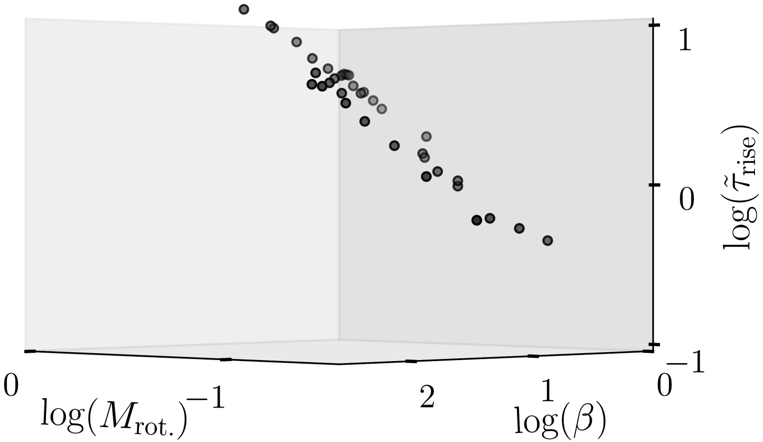
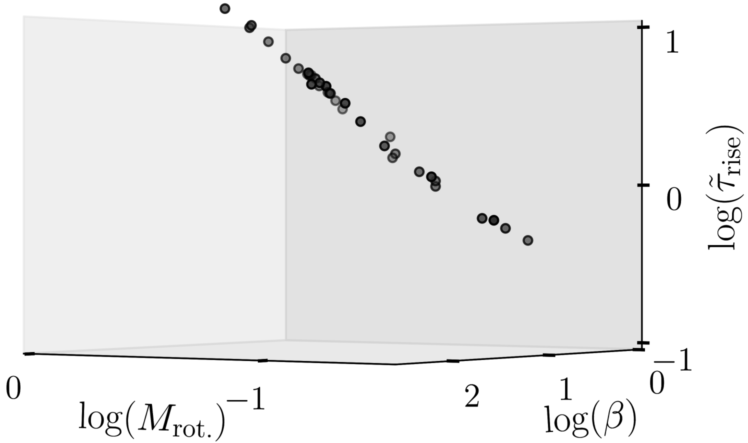
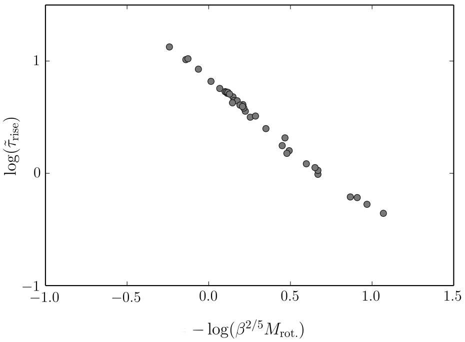


Looking for a scaling. (τ_{rise}, β, $\mathcal{M}_{\text{rot.}}$)



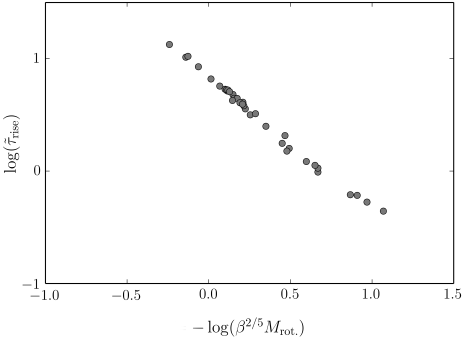
Reduction to 1D-function



The setup scales

- As a result of the scaling behavior of the setup we can reduce the relation to a 1D-function.

Reduction to 1D-function



The setup scales

- As a result of the scaling behavior of the setup we can reduce the relation to a 1D-function.
- **one independent variable**:

$$\left(\frac{1}{\beta}\right)^{2/5} \frac{1}{\mathcal{M}_{\mathrm{rot.}}}$$

What is the difference between 2D and 3D?

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- Not the geometry, but the azimuthal wavenumber (m) of the perturbation.
  - 2D: m=0 $\rightarrow$ axisymmetric rise.
  - 3D: m>0 $\rightarrow$ non-axisymmetric rise.

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- 3 series varying azimuthal wavenumber (m)

Series	Scaling parameter
m=0	$Ro^{\star}_{\mathrm{m}}$
m=4	$\frac{(v_{\mathrm{A}})^{0.92}(c_s)^{0.08}}{\varpi\Omega_0}$
m=8	$\frac{(v_{\mathrm{A}})^{0.85}(c_s)^{0.15}}{\varpi\Omega_0}$

Defining a *unified* scaling parameter.

- In 3D the properties of the rise of the magnetic flux tube do not scale with the magnetic Rossby numbers, so we had to define a more general scaling parameter. We call it Γ_α:

$$\Gamma_\alpha = \frac{(v_{\rm A})^\alpha \, (c_s)^{1-\alpha}}{\varpi \Omega_0}$$

- Where α depends on the azimuthal wavenumber (m) of the perturbation.

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- Where α depends on the azimuthal wavenumber (m) of the perturbation.
- In the case of m=0 $\rightarrow$ α=1 :

$$\Gamma_1 = \frac{v_{\mathrm{A}}}{\varpi \Omega_0} = Ro^{\star}_{\mathrm{m}}$$

There is no difference between 2D and 3D.

- Not the geometry, but the azimuthal wavenumber (m) of the perturbation.
  - 2D: m=0 $\rightarrow$ axisymmetric rise.
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Three series

- m = 0
- m = 4
- m = 8

One scaling

$$\Gamma_\alpha = \frac{(v_{\mathrm{A}})^\alpha (c_s)^{1-\alpha}}{\varpi \Omega_0}$$

Three α:

- 1
- 0.92
- 0.85

- α is a function of the azimuthal wavenumber of the perturbation

What do we learn about such a relation?

- The regime of the rise is driven by Γ_α
- We learn that **it is possible to compare our simulations with stellar objects**.
  - Even though the *numerical parameter space* is far away from the realistic one, Γ_α drives the regime, hence a simulation with the solar Γ_α can be compared with the sun.

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- And what about the dynamo?

Application: the Babcock-Leighton dynamo

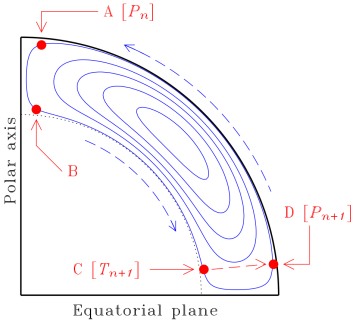


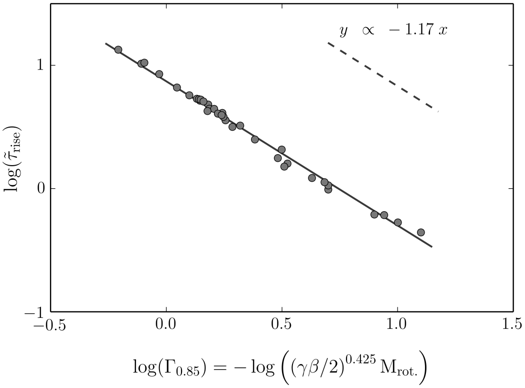
Figure : A sketch of the Babcock-Leighton dynamo mechanism. Picture from Charbonneau (2010).

- The BL-Dynamo needs a mechanism to transport magnetic flux from the bottom of the CZ to the surface. The traditional mechanism is magnetic buoyancy under the form of flux tubes.
- Usually people consider the rise to be instantaneous. Is it really?
- From our simulations we can constrain the rise.

Evolution of the relative rising time with Γ_α

- For an azimuthal wavenumber $m = 8$

$$\tilde{\tau}_{\text{rise}} \propto (\Gamma_{0.85})^{-1.17}$$



Future work: implementing a delayed BL-dynamo (testing phase)

- In a similar manner than Jouve et al. (2010) we are introduced a delay in the α-effect.
- From our results we can constrain the delay (for m=8):

$$\tau_{\mathrm{B}} = \left(\frac{\tau_0}{\tau_{\mathrm{diff}}}\right) \ \sin(\theta) \ (\mathcal{M}_{\mathrm{rot.}})^{0.17} \ \left(\frac{AB_\phi}{B_{\mathrm{Eq}}}\right)^{-1}$$

- We want to study the possible limit of the BL-Dynamo. Fast rotating stars has:
  - shorter cycles
  - longer rising time

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- We want to study the possible limit of the BL-Dynamo. Fast rotating stars has:
  - shorter cycles
  - longer rising time
- What would happen if the rising time becomes of the same order of magnitude than the cycle?

Discussion and Conclusion

- We conduct numerical experiments of compressible magnetic flux tubes, which were able to reproduce results found in the literature.
- In 3D, the setup scales in a different manner than in 2D.
- We designed a general scaling parameter unifying former and current results:

$$\Gamma_\alpha = \frac{\left(v_{\rm A}\right)^{\alpha} \left(c_s\right)^{1-\alpha}}{\varpi\Omega_0}$$

- The setup scales. Therefore, even though the numerical domain is different from the realistic one, the simulations are in the solar regime.
- We could compute the rising time scale needed to constrain the BL-dynamo.
- We include this rising time as a delay in the α effect for a BL-dynamo.