Dynamo bifurcations and saturations mechanisms in global geodynamo simulations

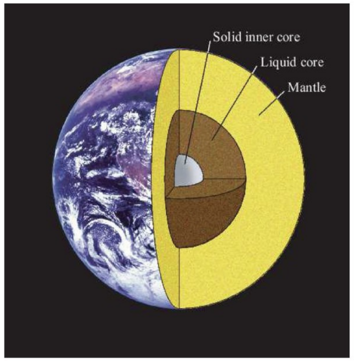
Ludovic Petitdemange
ludovic@lra.ens.fr
Laboratoire de RadioAstronomie
LRA-ENS LERMA Paris

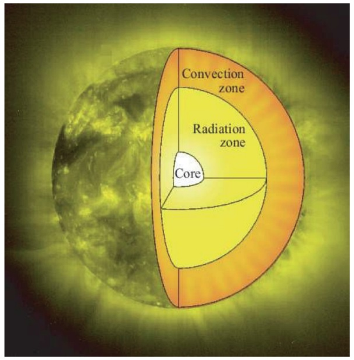
Collaborators :
Martin Schrinner
Emmanuel Dormy
Raphael Raynaud

Stellar and Planetary dynamos   Göttingen 2015

contents

- Introduction and motivations
- Standard geodynamo model
- Dynamo bifurcations in the parameter space
- Saturation mechanism for subcritical dynamos
- Attempts to deduce scaling laws





Dynamo Models

• Conducting Boussinesq fluid in a rotating spherical shell.

• Convection driven by fixed temperature gradient between inner and outer shell

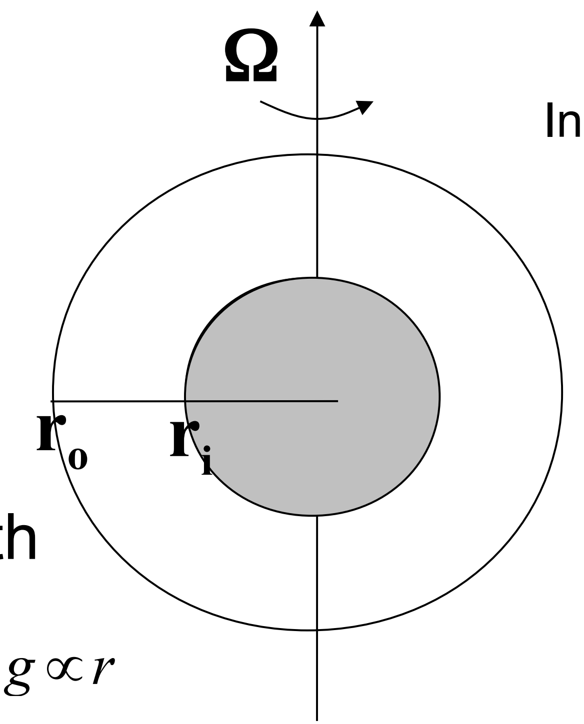
•Systematic parameters survey:

- Different initial conditions
- Dimensionless parameters
- B.C. : Stress-Free or Rigid

• MHD-code:

PaRoDy (Dormy et al. 1998) with

homogenous mass distribution $g \propto r$



Schrinner, Petitdemange, Dormy (2011,2012)

MHD equations

$$(\mathrm{E}/\mathrm{Pm})\ [\partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u}] = -\boldsymbol{\nabla}\pi + \mathrm{E}\,\Delta\mathbf{u} - 2\mathbf{e}_z \times \mathbf{u} + \mathrm{Ra}\,T\,\mathbf{r} + (\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B}\,,$$

$$\partial_t \mathbf{B} = \boldsymbol{\nabla}\times(\mathbf{u}\times\mathbf{B}) + \Delta\mathbf{B}\,,$$

$$\partial_t T + (\mathbf{u}\cdot\boldsymbol{\nabla})T = \mathrm{q}\Delta T\,,$$

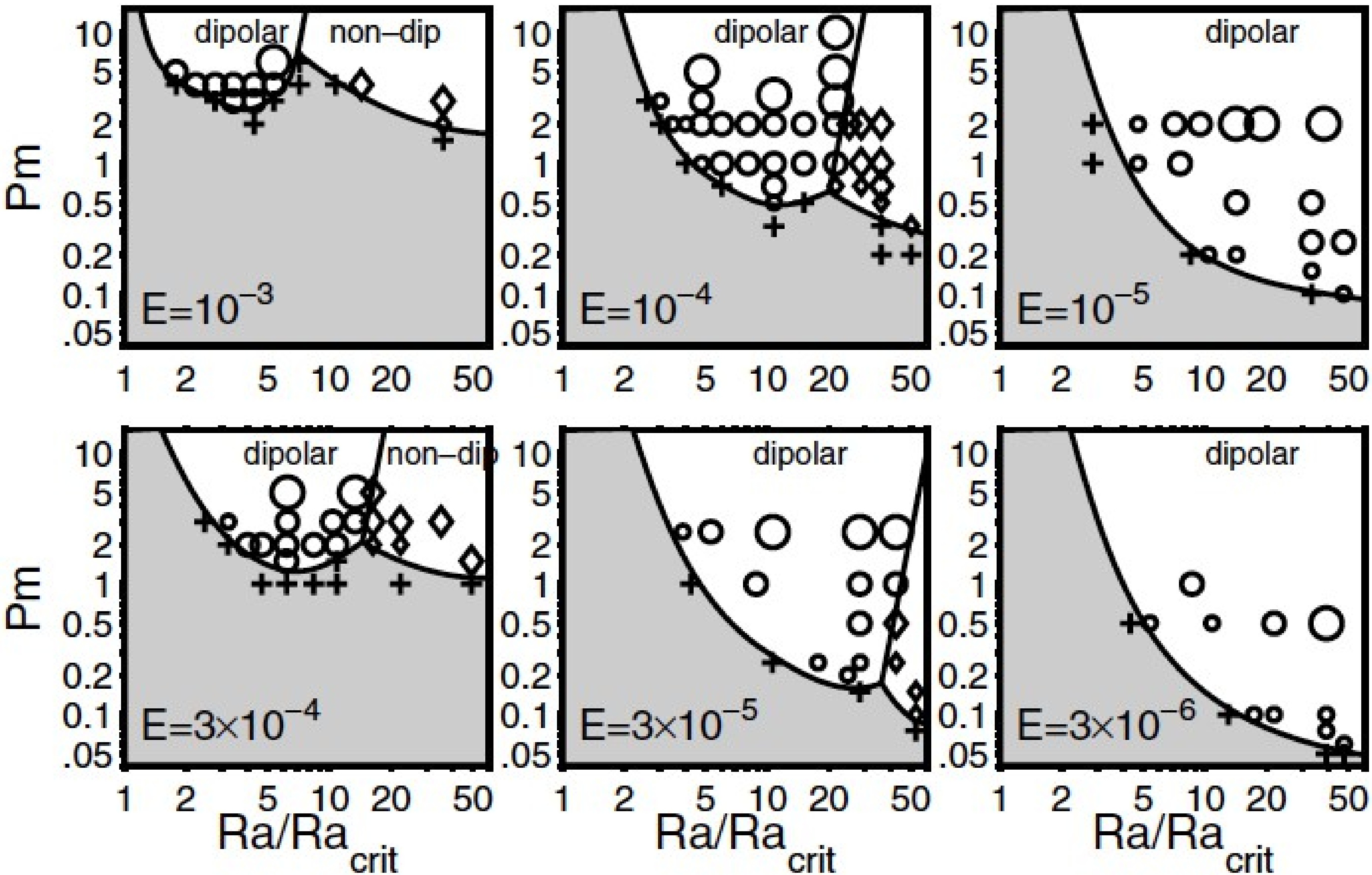
$$\boldsymbol{\nabla}\cdot\mathbf{u} = \boldsymbol{\nabla}\cdot\mathbf{B} = 0\,.$$

$$E/Pm = E_\eta = \frac{\eta}{\Omega\mathcal{L}^2}\,,\quad E = \frac{\nu}{\Omega\mathcal{L}^2}\,,\quad q = \frac{\kappa}{\eta}\,.$$

$\sim 10^{-9}$ $\sim 10^{-15}$ $\sim 10^{-6}$

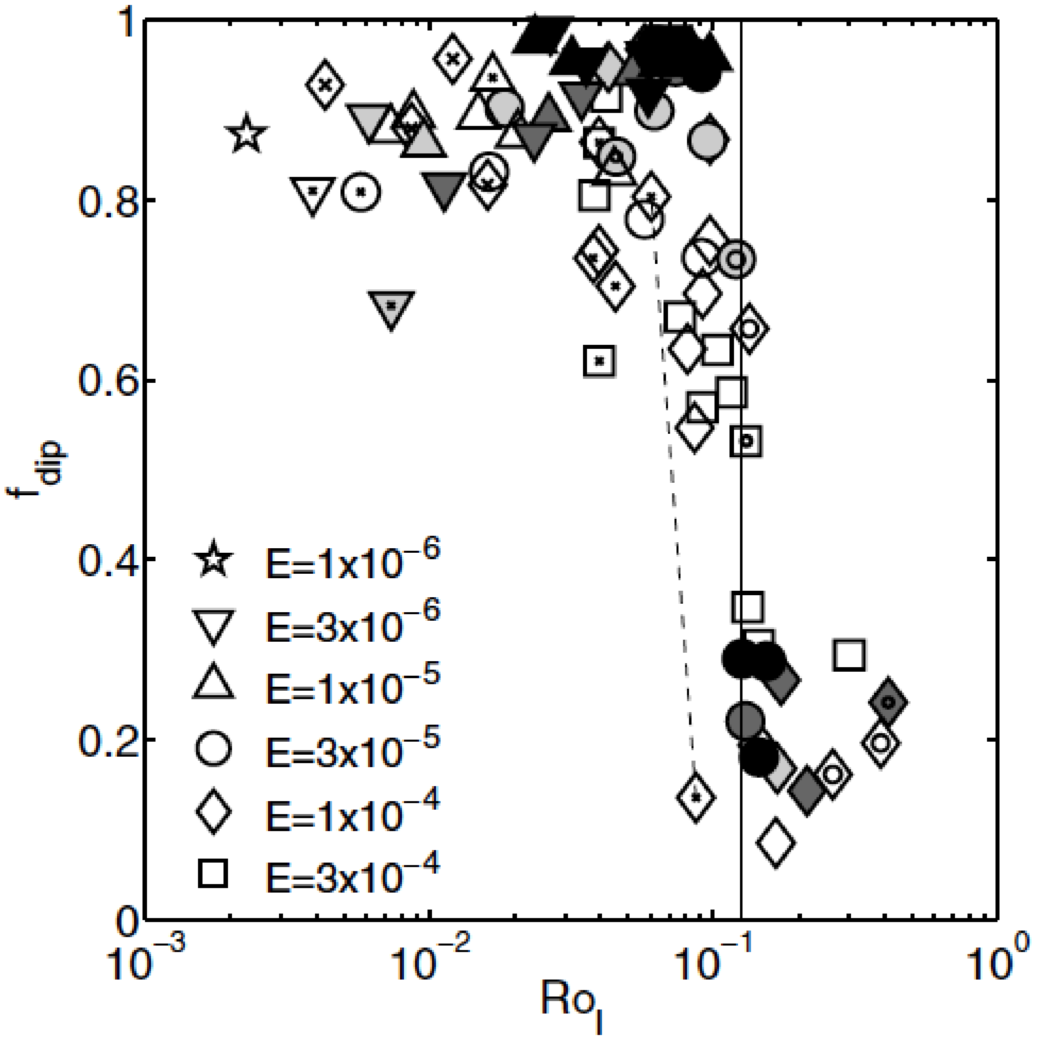
$\geq 10^{-6}$ $\geq 10^{-6}$ ≥ 0.1

Dynamo regimes in geodynamo simulations



Christensen & Aubert 2006

Two distinct regimes in geodynamo simulations



Christensen & Aubert, 2006

$$\bar{\ell}_u = \frac{\sum \ell \langle \mathbf{u}_\ell \cdot \mathbf{u}_\ell \rangle}{2E_{\mathrm{kin}}},$$

$$Ro_\ell = Ro\frac{\bar{\ell}_u}{\pi}.$$

Dipole field strenght f_{dip}: time-average ratio on the outer shell boudary of the mean dipole field strength to the field strength in harmonic degrees l=1-12.

Test-field method

• Perform MHD-simulation.

• « Measure » the mean electromotive force $\mathcal{E}$ generated by the action of the velocity field on certain test-fields.

• Extract coefficients out of expansion of $\mathcal{E}$,

$$\mathcal{E}_{\kappa}^{(i)} = \alpha_{\kappa\lambda} B_{T\lambda} + \beta_{\kappa\lambda\mu} \frac{\partial B_{T\lambda}}{\partial x_{\mu}}$$

Schrinner et al. 2005, 2007

Brandenburg, Rädler, Schrinner 2008

Tilgner, Brandenburg 2008

Rädler, Brandenburg 2009

…

Eigenvalue Problem

$$
\begin{aligned}
\lambda \boldsymbol{B} &= \nabla \times D\boldsymbol{B} \\
D\boldsymbol{B} &= \boldsymbol{u} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}
\end{aligned}
$$

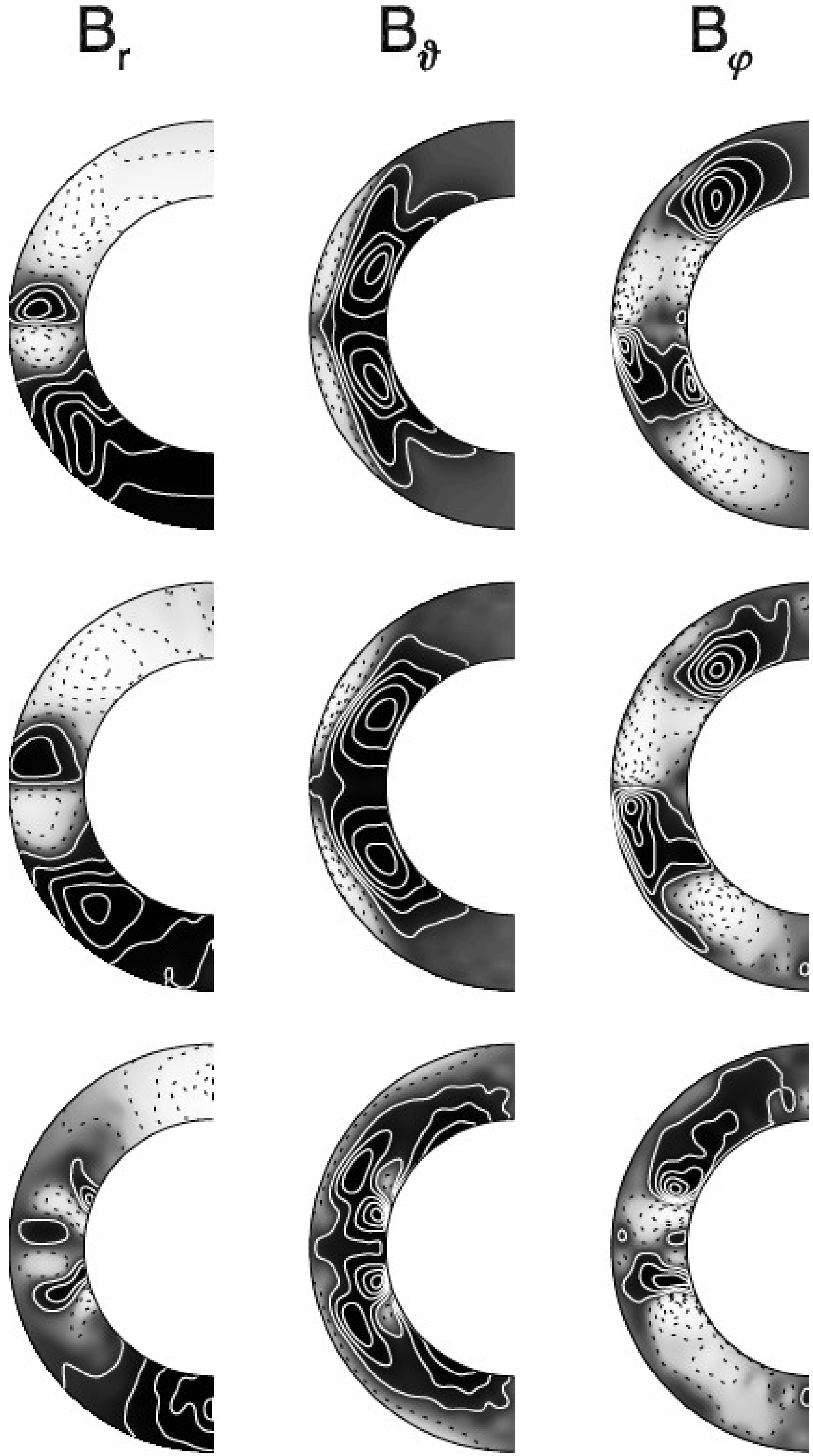
Averaging in azimuth:

$$
D\boldsymbol{B} = \boldsymbol{V} \times \boldsymbol{B} + \boldsymbol{\alpha} \cdot \boldsymbol{B} - \boldsymbol{\beta} : \nabla(\boldsymbol{B}) - \eta \nabla \times \boldsymbol{B}
$$

α,β : Tensors of second and third rank respectively, depending on the velocity field.

They are determined with the help of the test-field method.

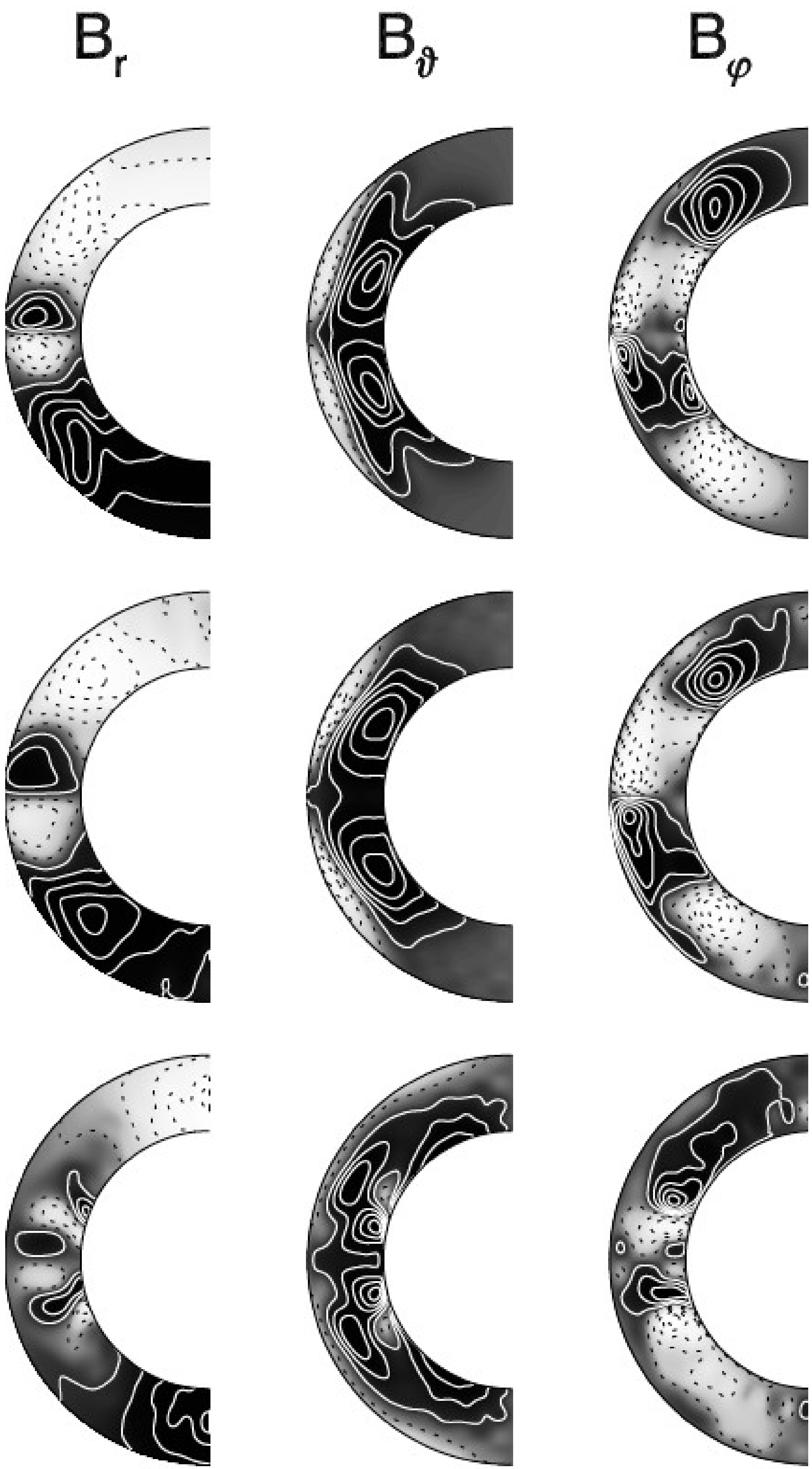
(e.g. Schrinner et al. 2005, 2007, Schrinner et al. 2010)



Time-average magnetic components from DNS

Eigen mode from mean-field simulation with dynamo coefficient obtained using the test-field method.

Mean-field calculation without Υ effect



Time-average magnetic components from DNS

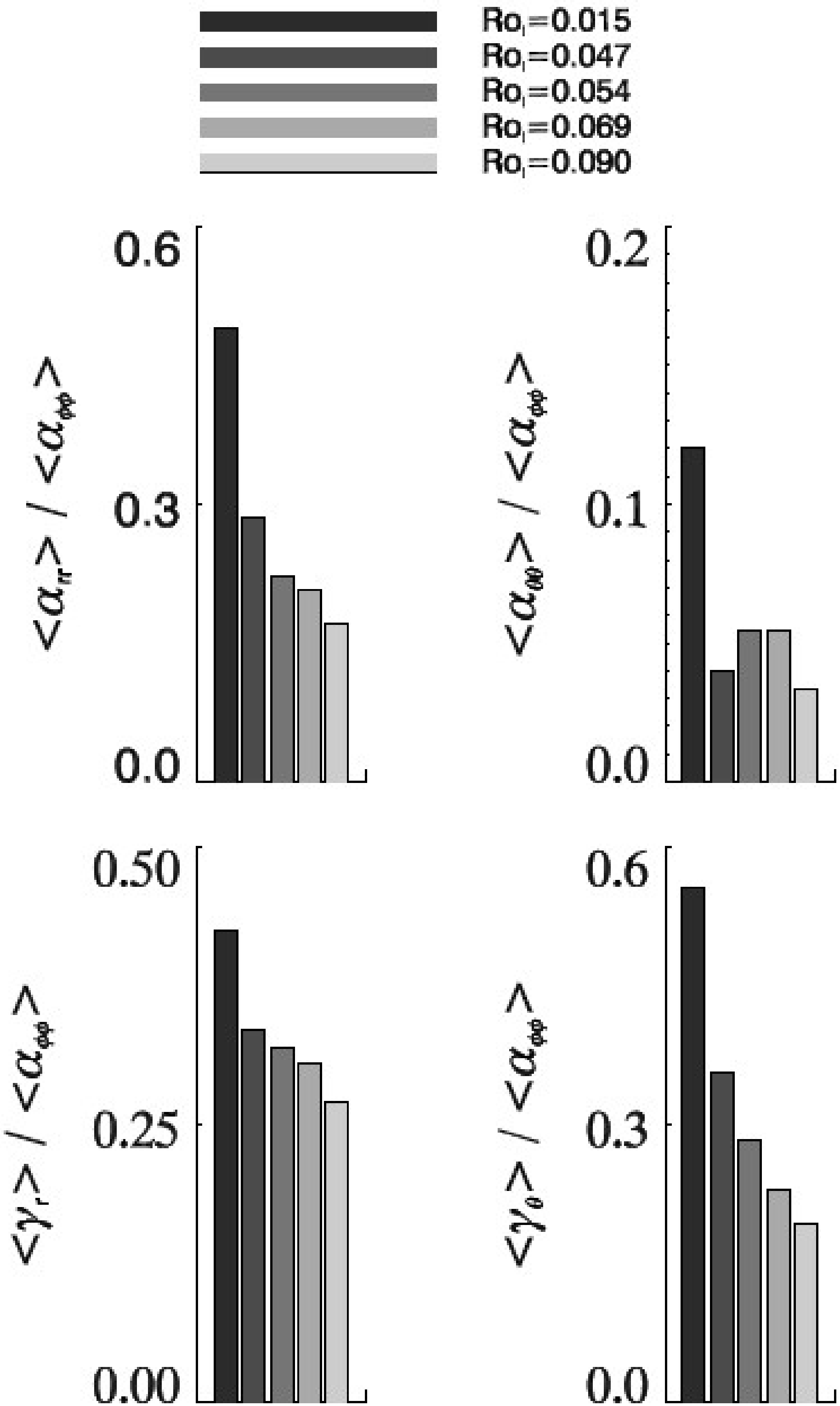
Eigen mode from mean-field simulation with dynamo coefficient obtained using the test-field method.

Mean-field calculation without Υ effect

Relaxing the cylindrical constraint in a spherical shell

Dipole collapse induced by inertia

Schrinner, Petitdemange, Dormy, ApJ 2012.



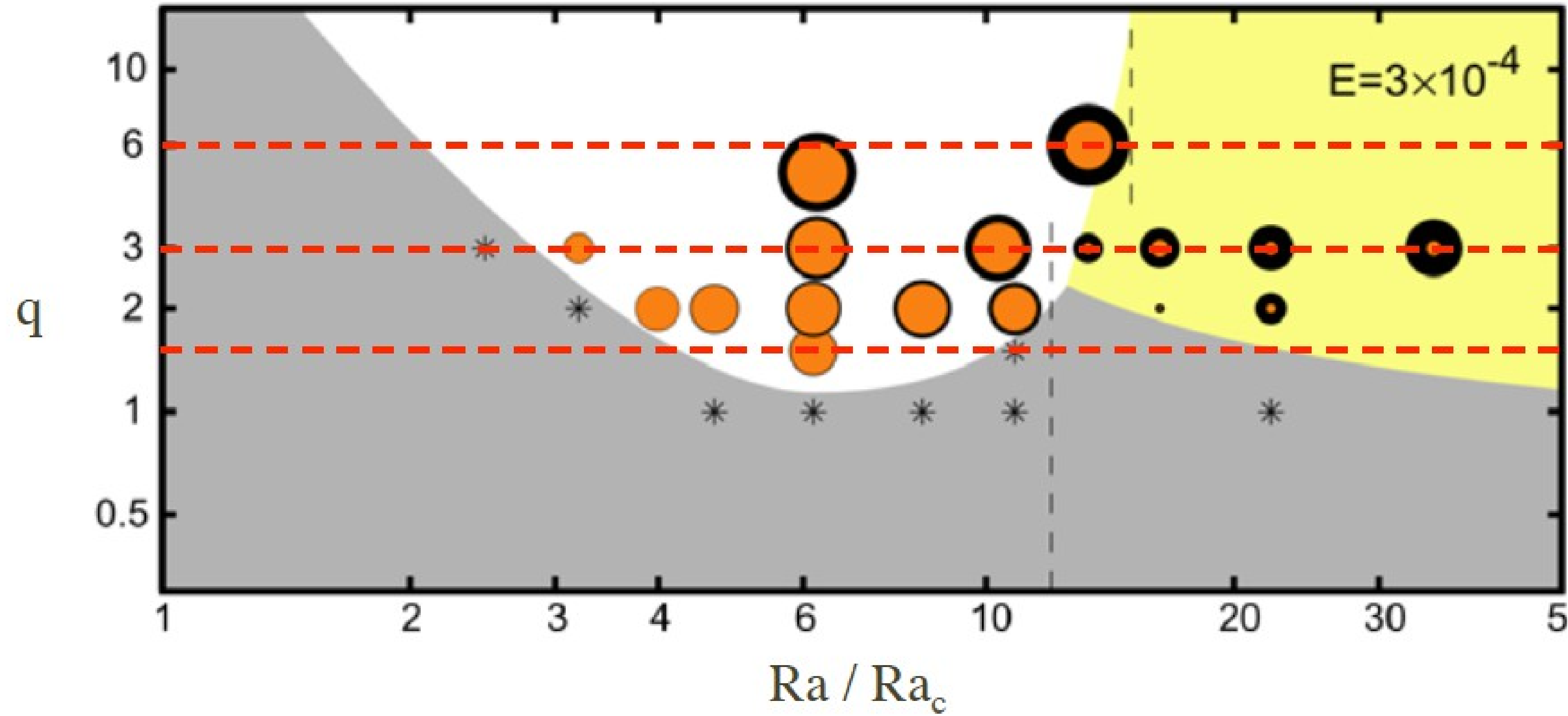
Effects of relaxing the cylindrical constraint on dynamo coefficients by increasing RoI

$\alpha_{\varphi\varphi}$ is unchanger

The ϒ effect is important for α^2 dynamo (dipolar branch) and it is present only if the cylindrical constraint of the flow is efficient.

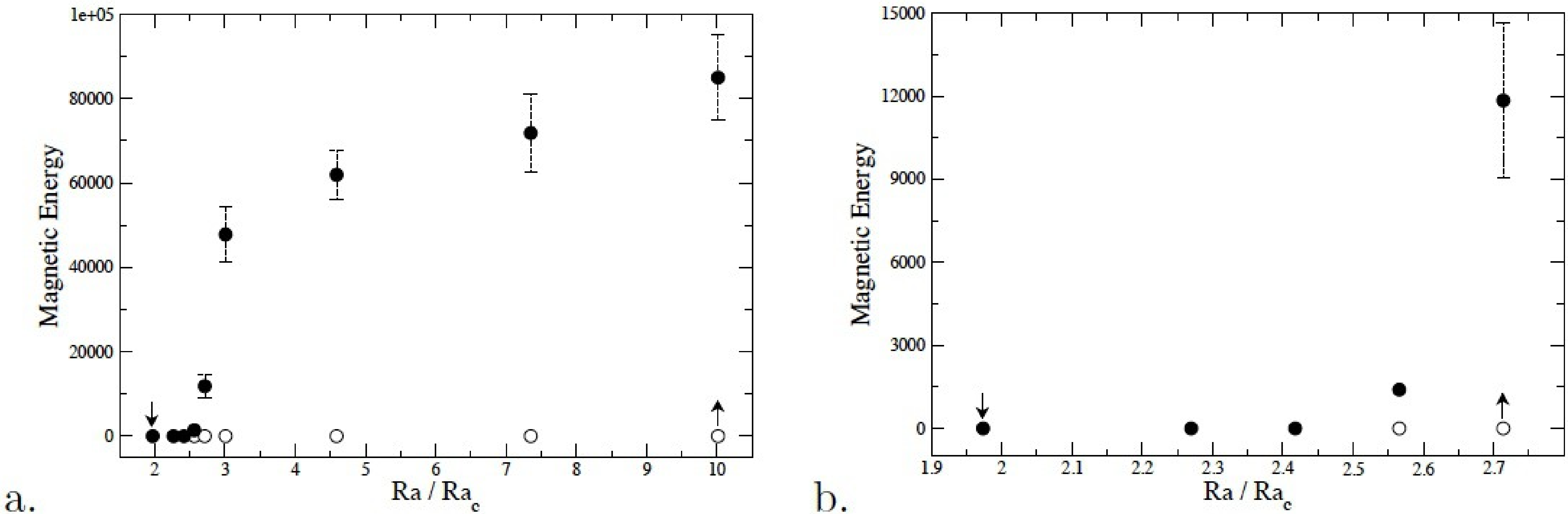
In square box dynamo simulations : No axial dipole field and no ϒ effect Because no symmetry constraint. Tilgner 2012, Guervilly's talk.

Dynamo bifurcations



Control parameter : **Ra**

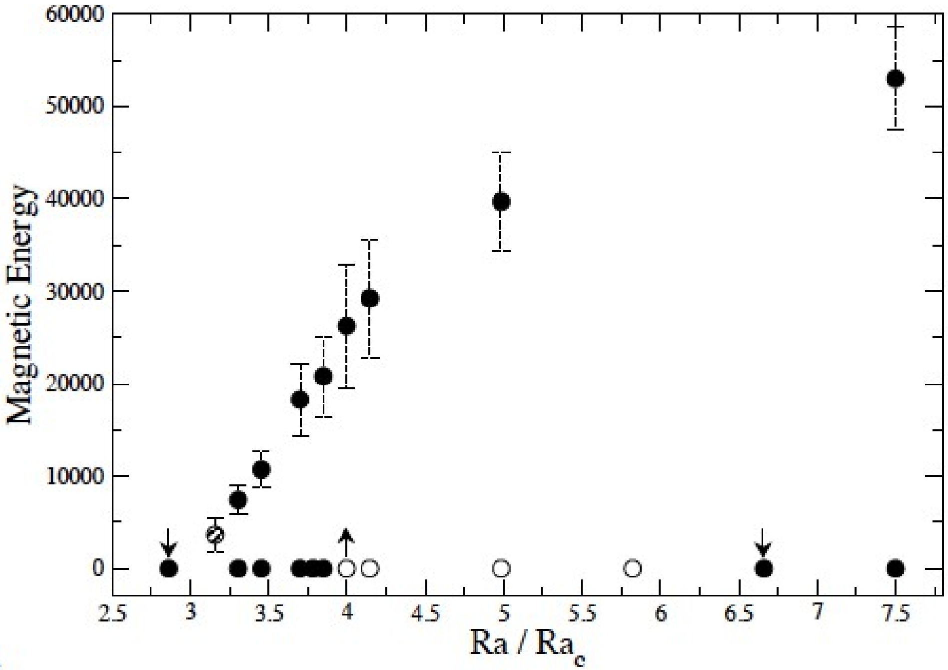
Supercritical dynamo bifurcation



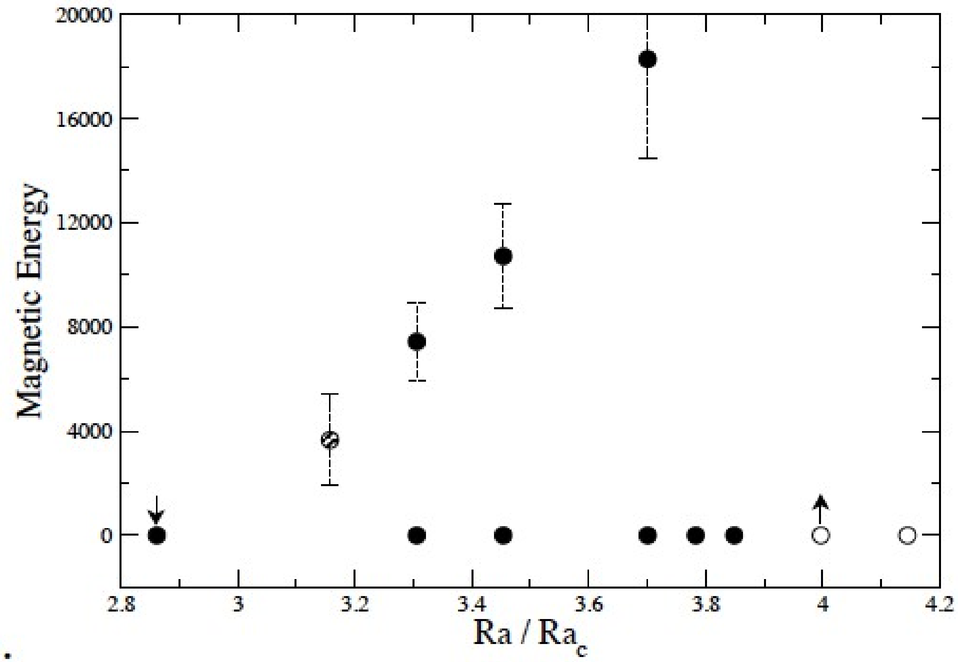
E=3.10^{-4} ; Pr=1 ; Pm=6

Morin & Dormy 2009

Subcritical dynamo bifurcation



a.

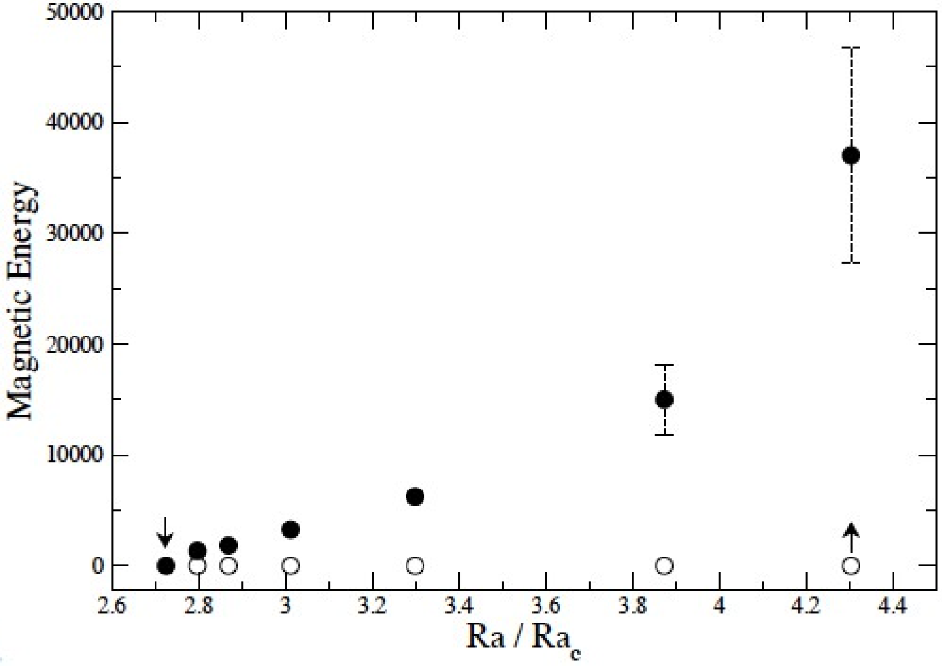


b.

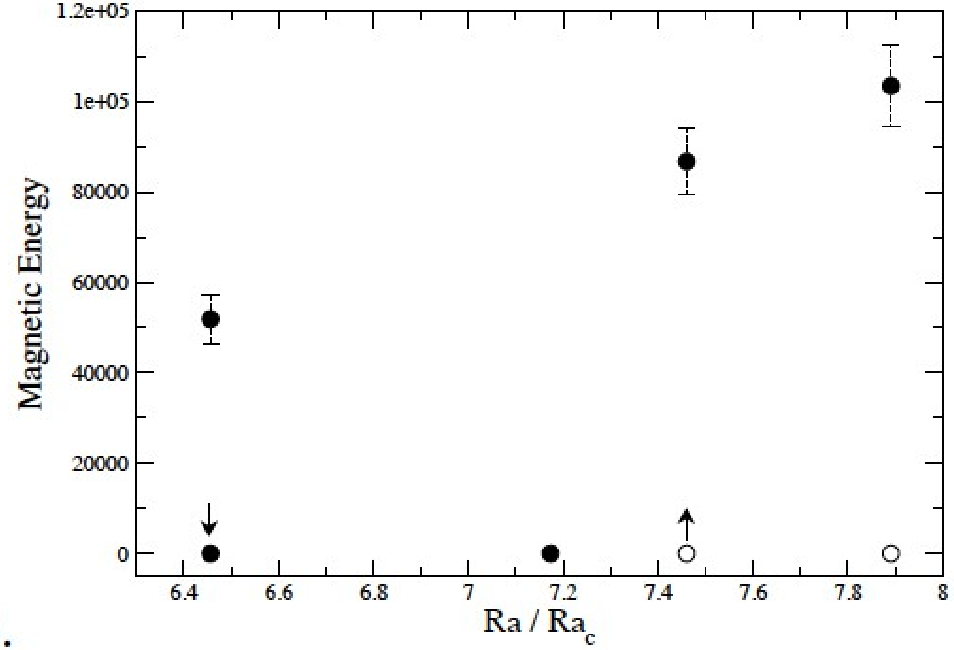
E=3.10^{-4} ; Pr=1 ; Pm=3

Morin & Dormy 2009

Dynamo bifurcations with changing Ek



a.

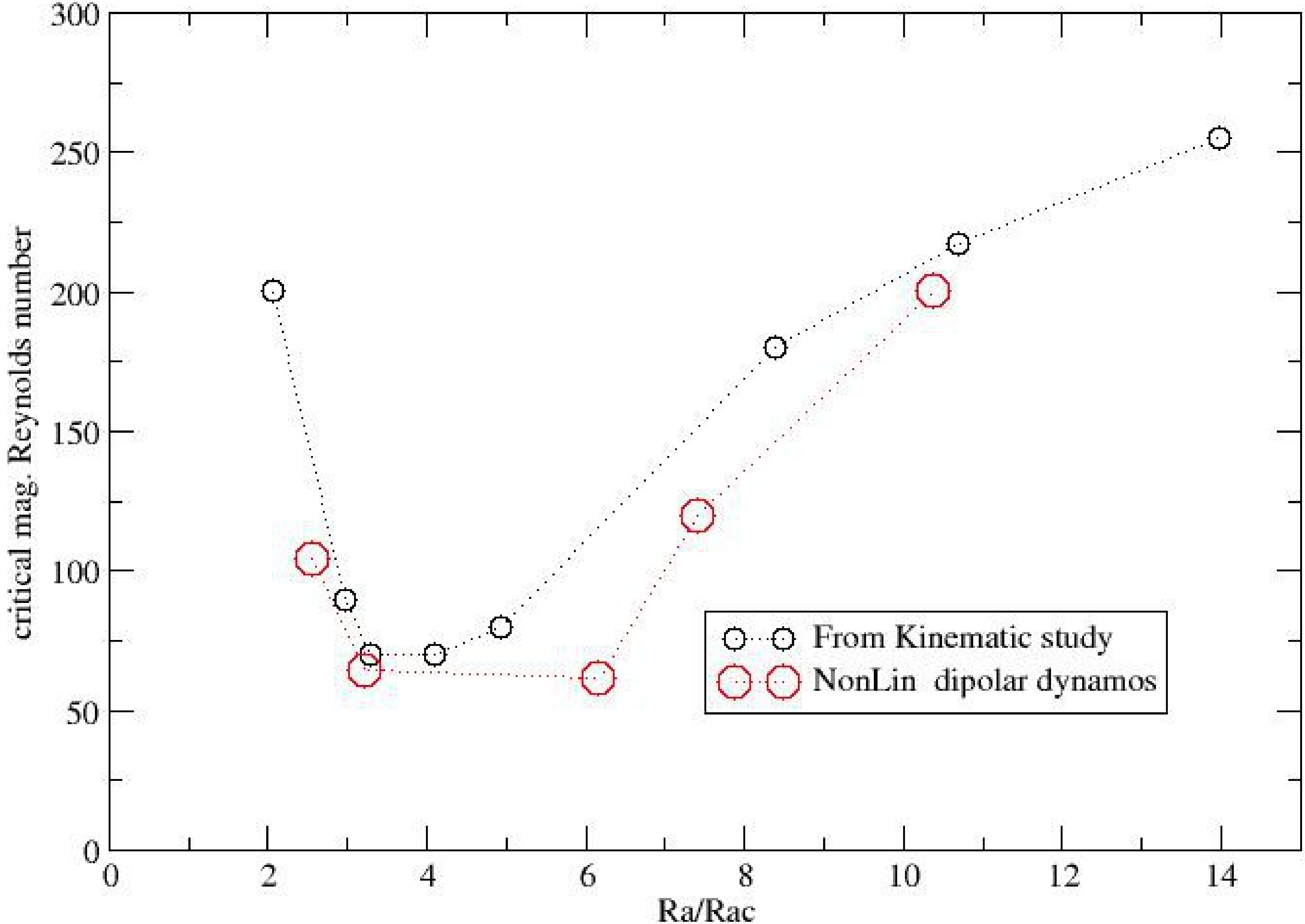


b.

E=1.10^{-4} ; Pr=1 ; Pm=3 and Pm=0.67

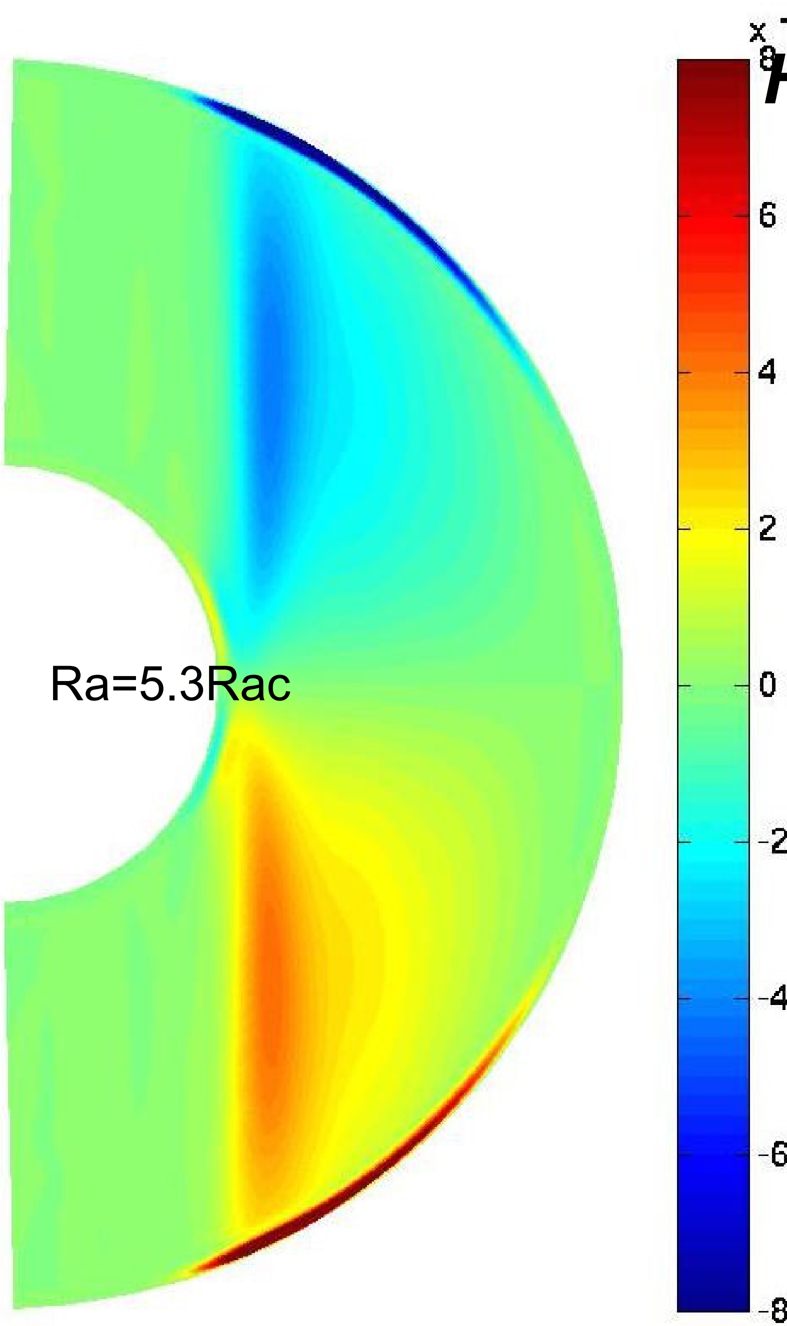
Morin & Dormy 2009

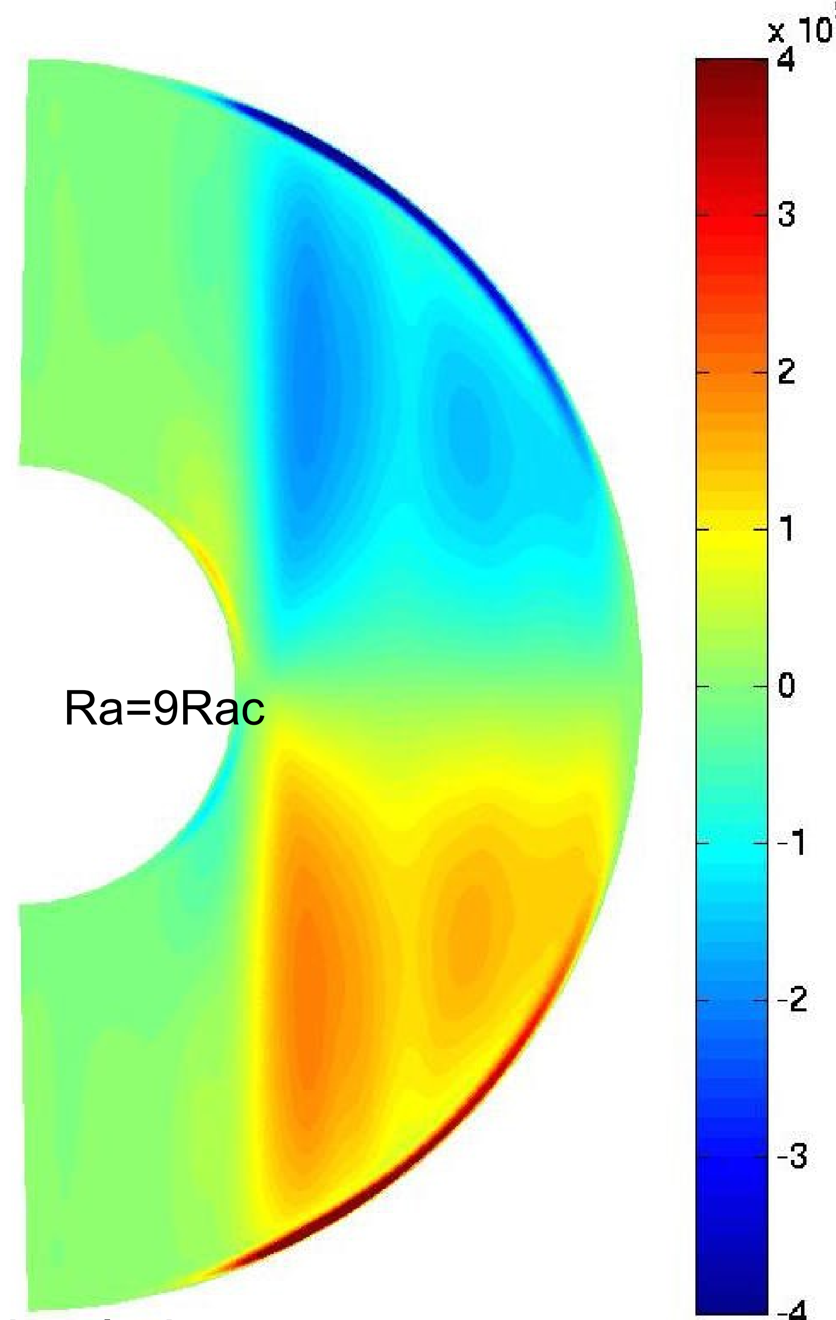
Kinematic study in ignoring Lorentz force



Subcritical dynamo : magnetic field effect

Helicity

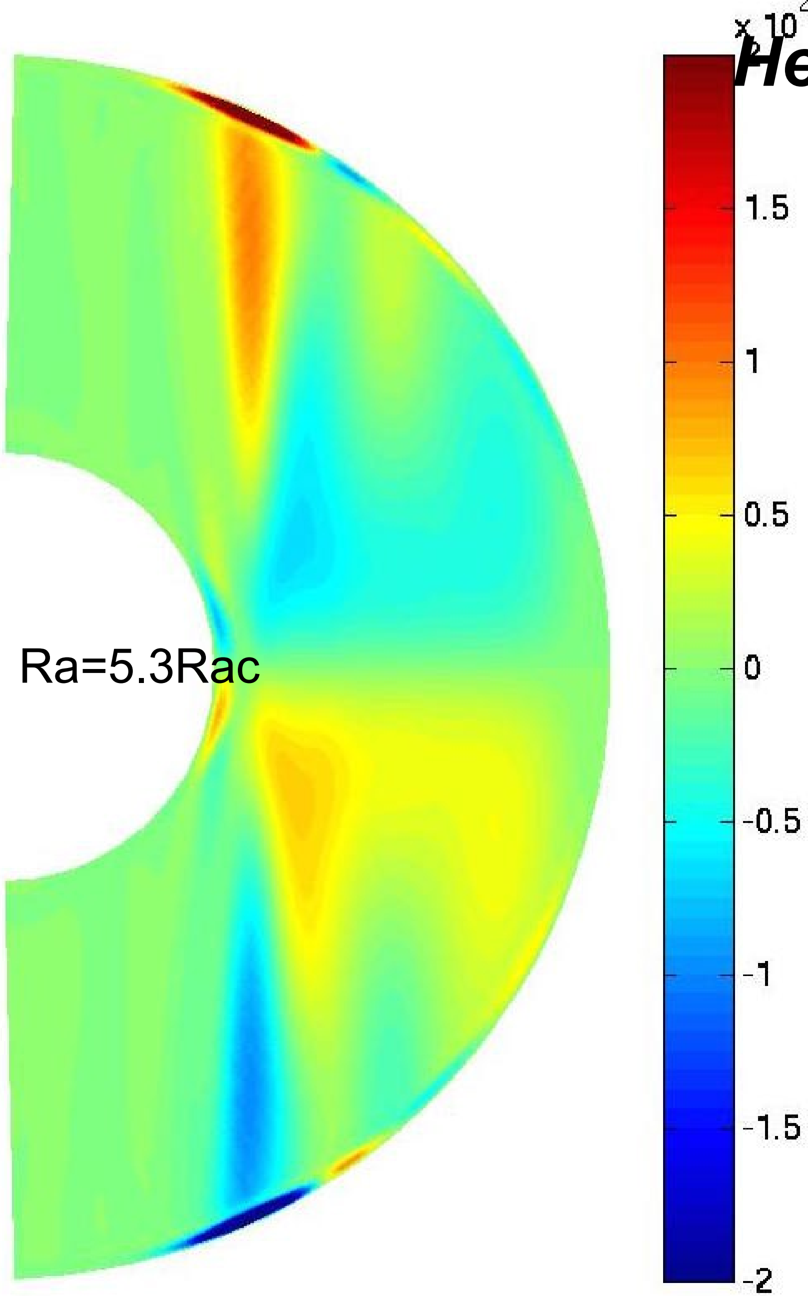


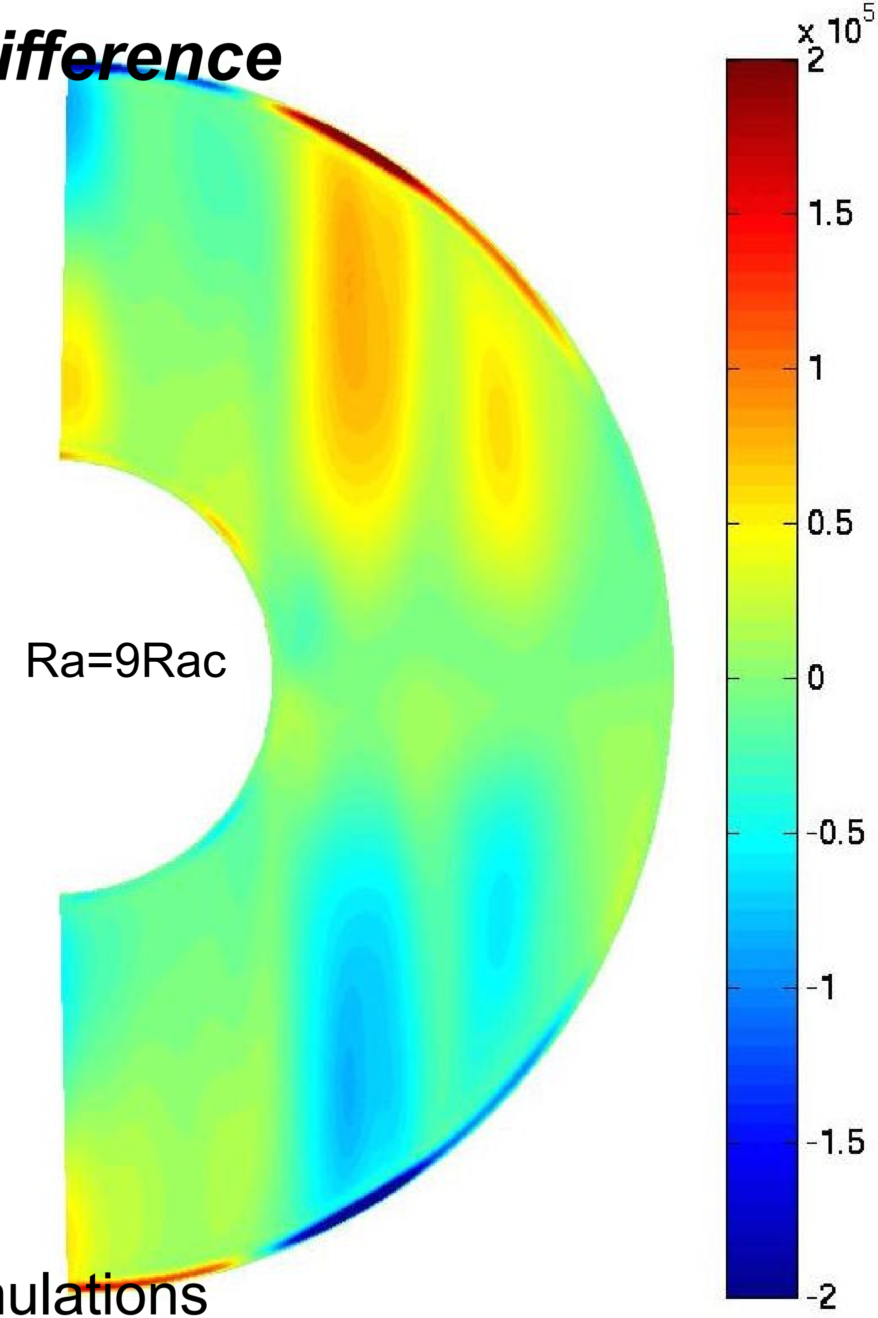


Hydro simulations

Subcritical dynamo : magnetic field effect

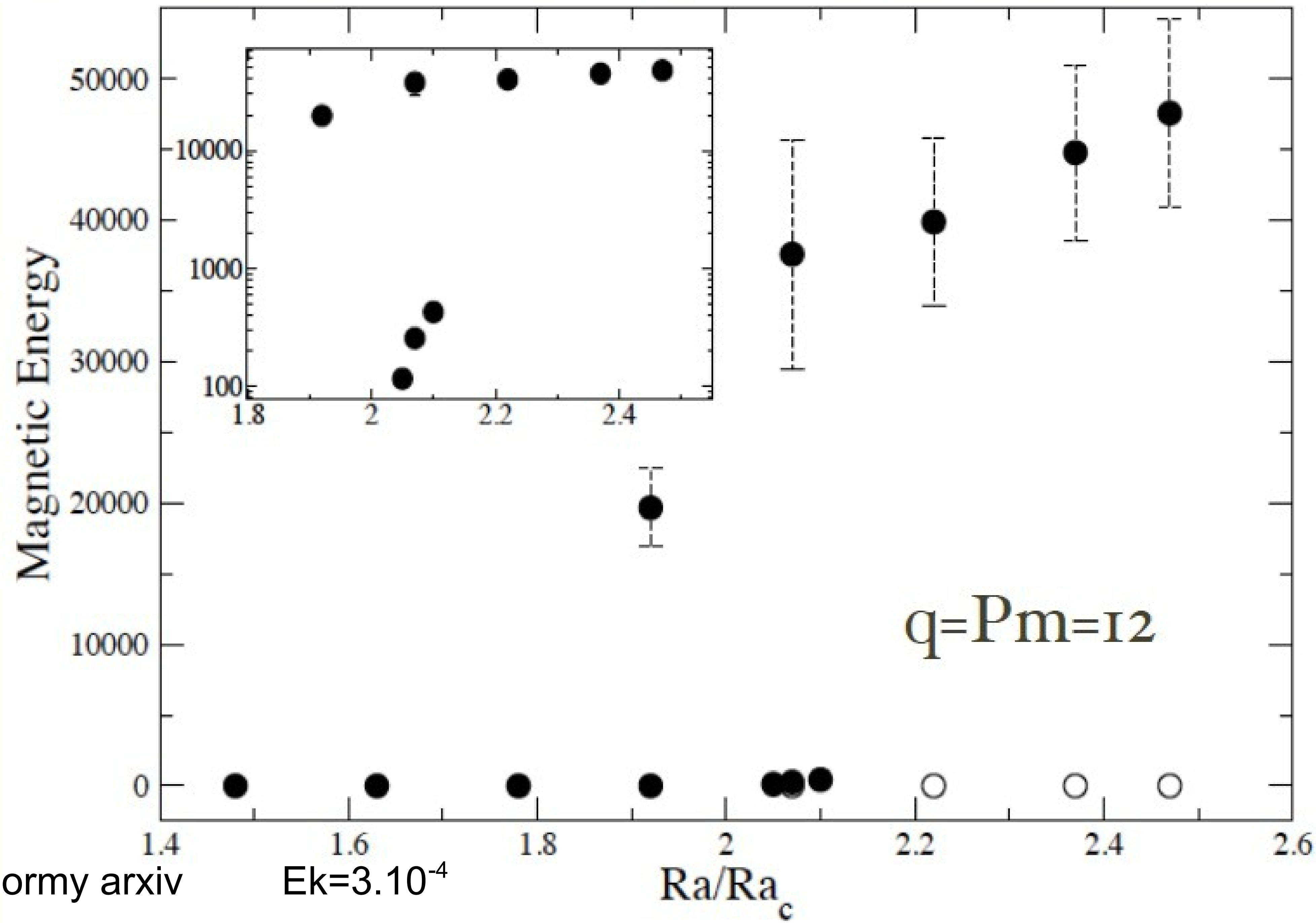
Helicity difference





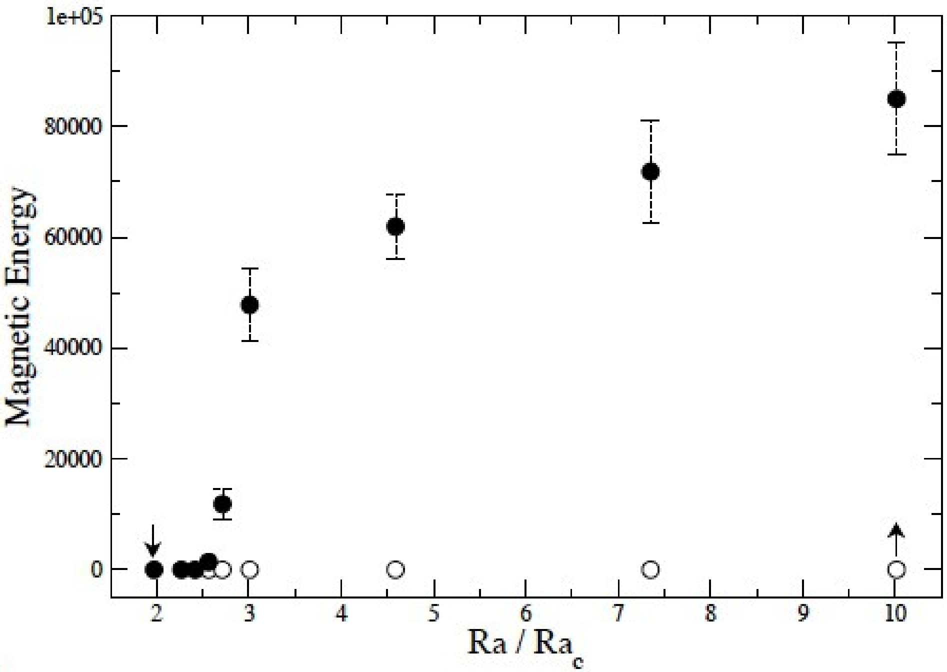
MHD simulations

Weak and Strong field branches

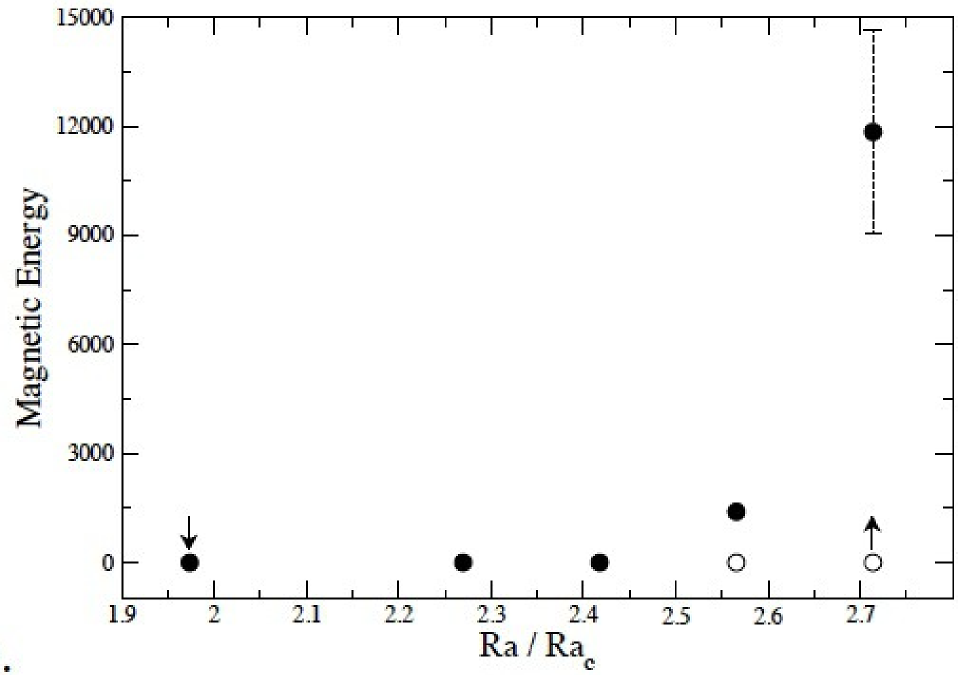


Dormy arxiv Ek=3.10^{-4}

Supercritical dynamo bifurcation



a.

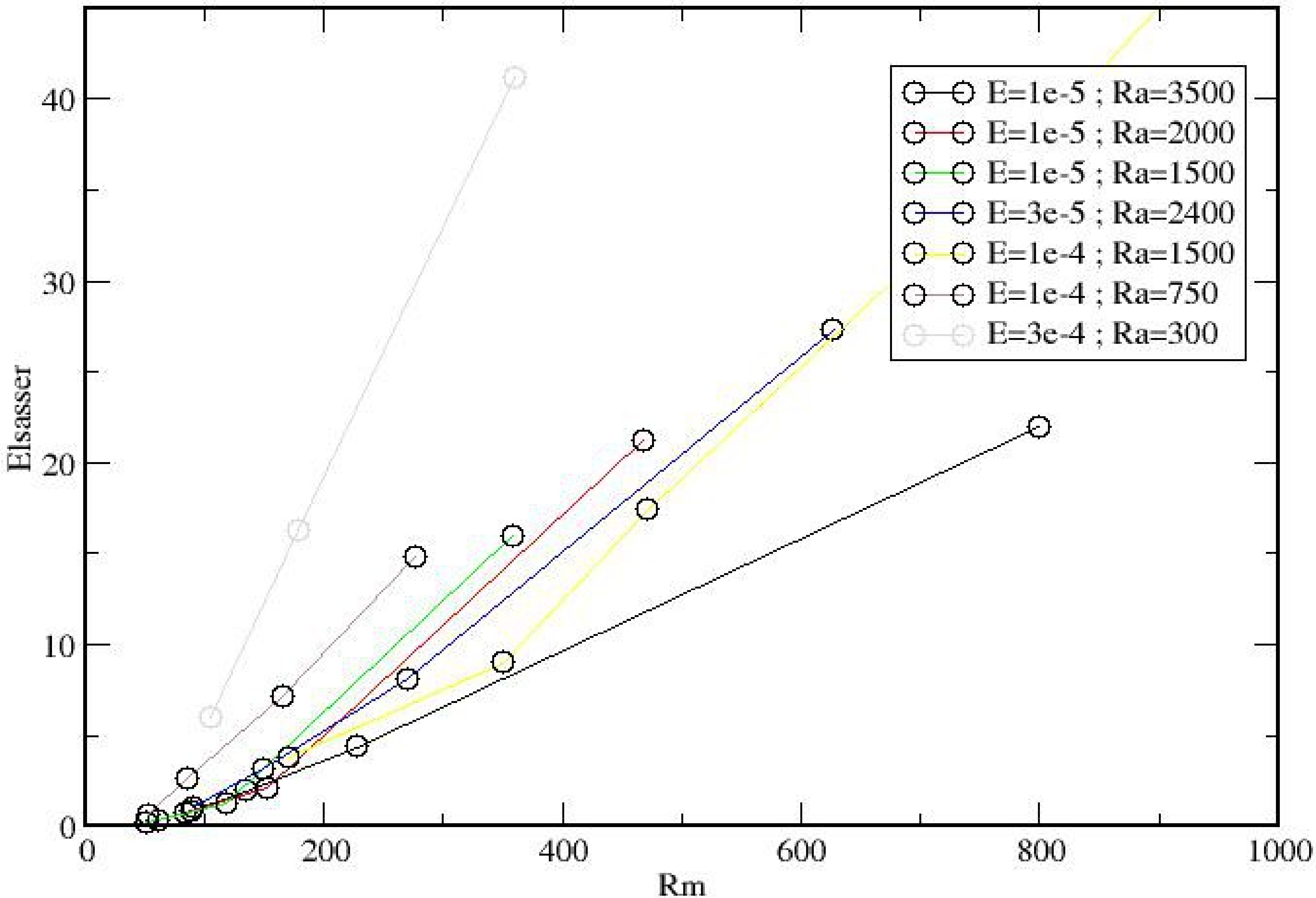


b.

$E=3.10^{-4}$; Pr=1 ; Pm=6

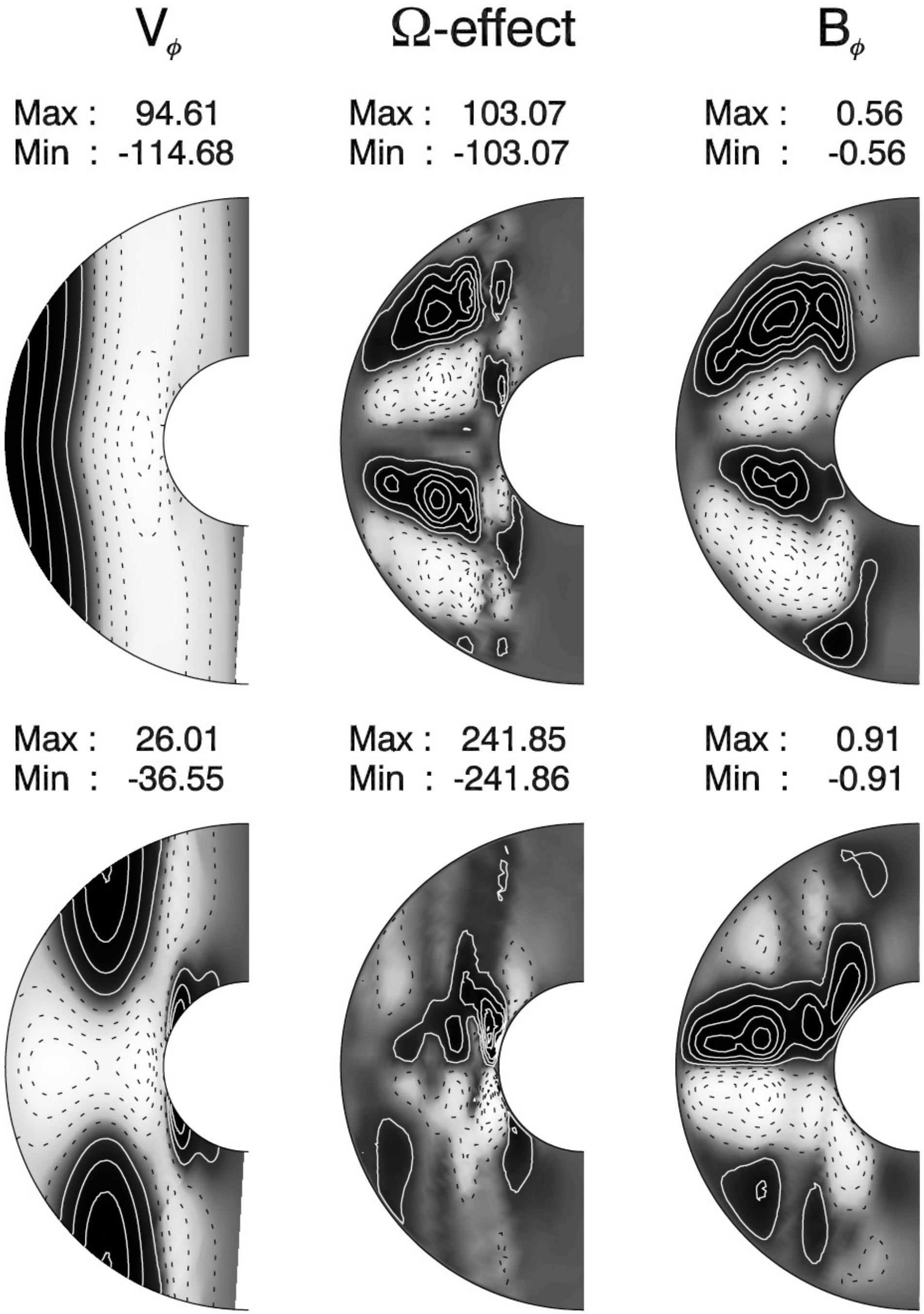
Morin & Dormy 2009

Control parameter : Pm



Data from Christensen & Aubert 2006, Schrinner et al (2012) and King 2010

Bistability SF/SF B.C.



multipolar field branch

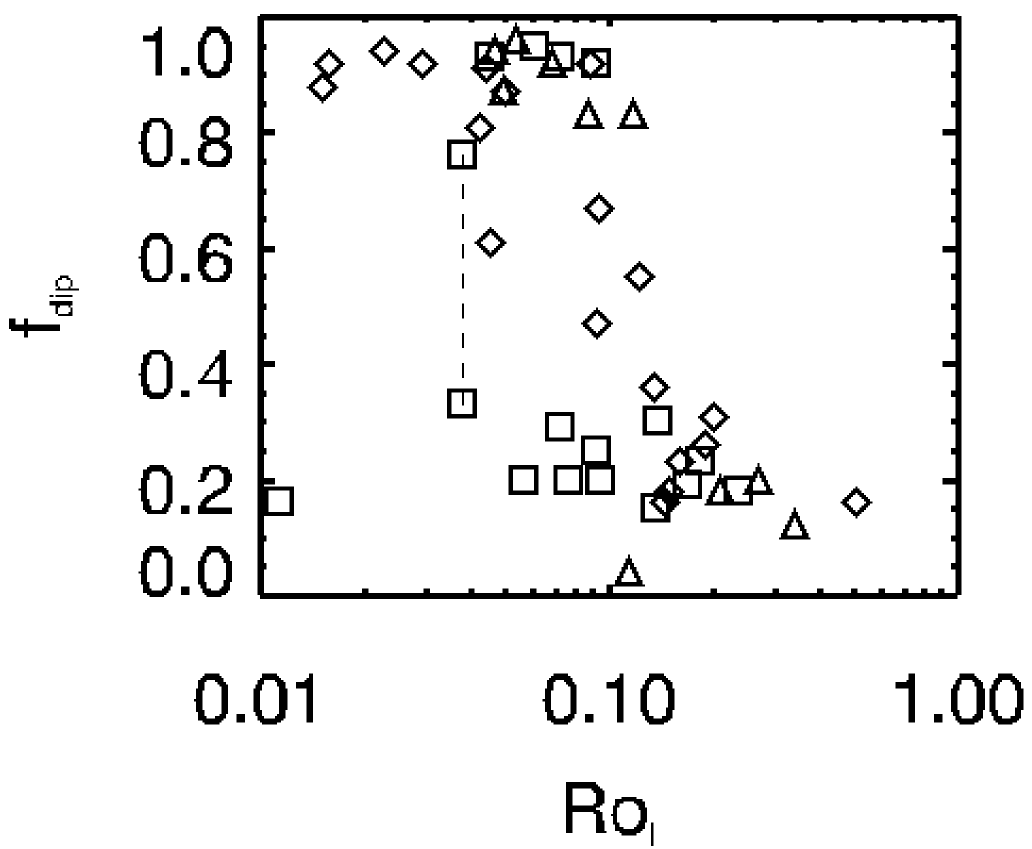
Oscillatory dynamos

Dipolar field branch

Dipolar and stationary

Schrinner, Petitdemange, Dormy (2012) ApJ

Bistability induced by large-scale differential rotation

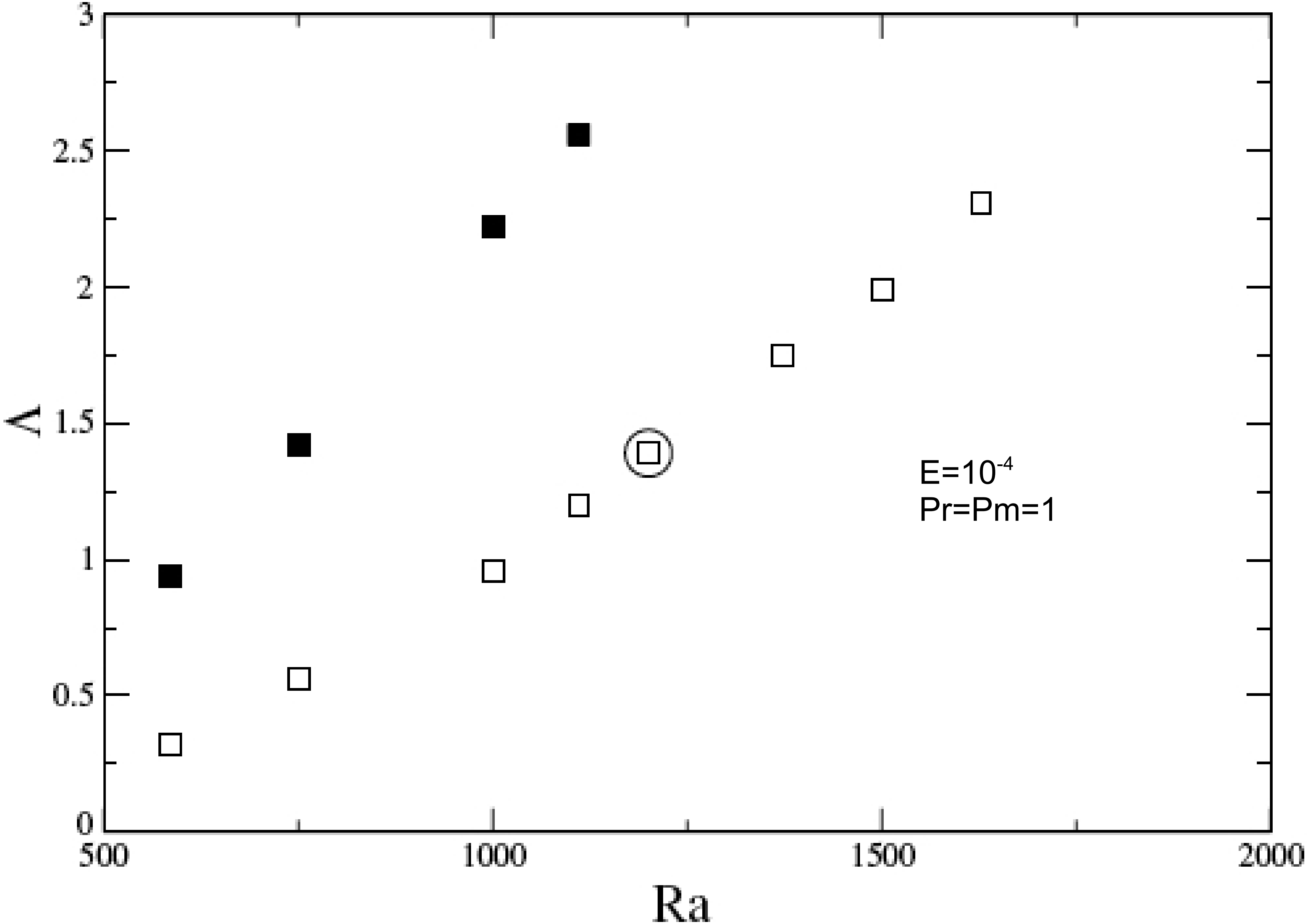


Boundary conditions

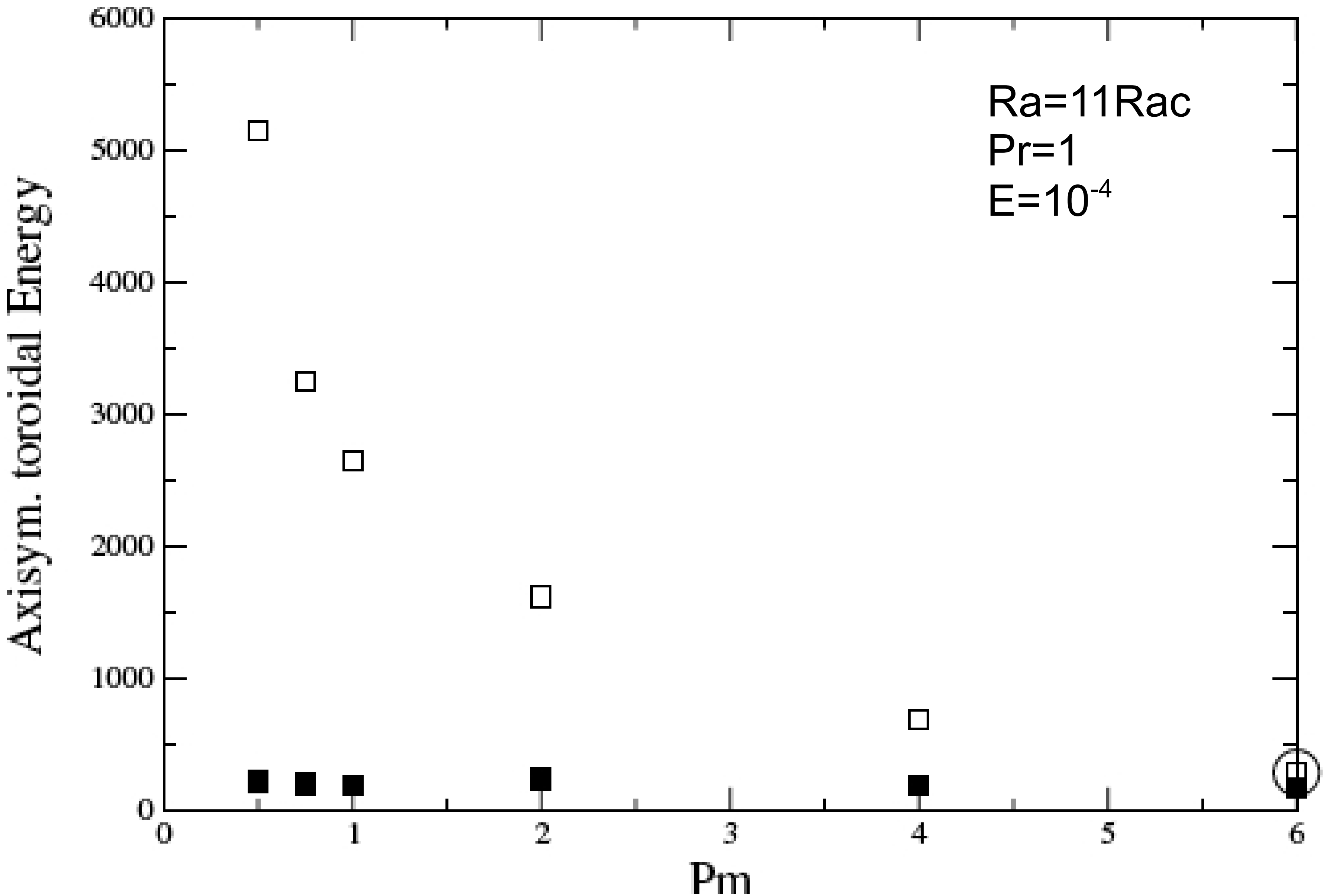
Δ Rigid/Stress-Free
□ Stress-Free/Stress-Free (SF/SF)
◊ Rigid/Rigid

Schrinner *et al* ApJ 2012
See Appendix for a discussion
On the Rol definition !

Dynamo branches with SF-SF boundary conditions



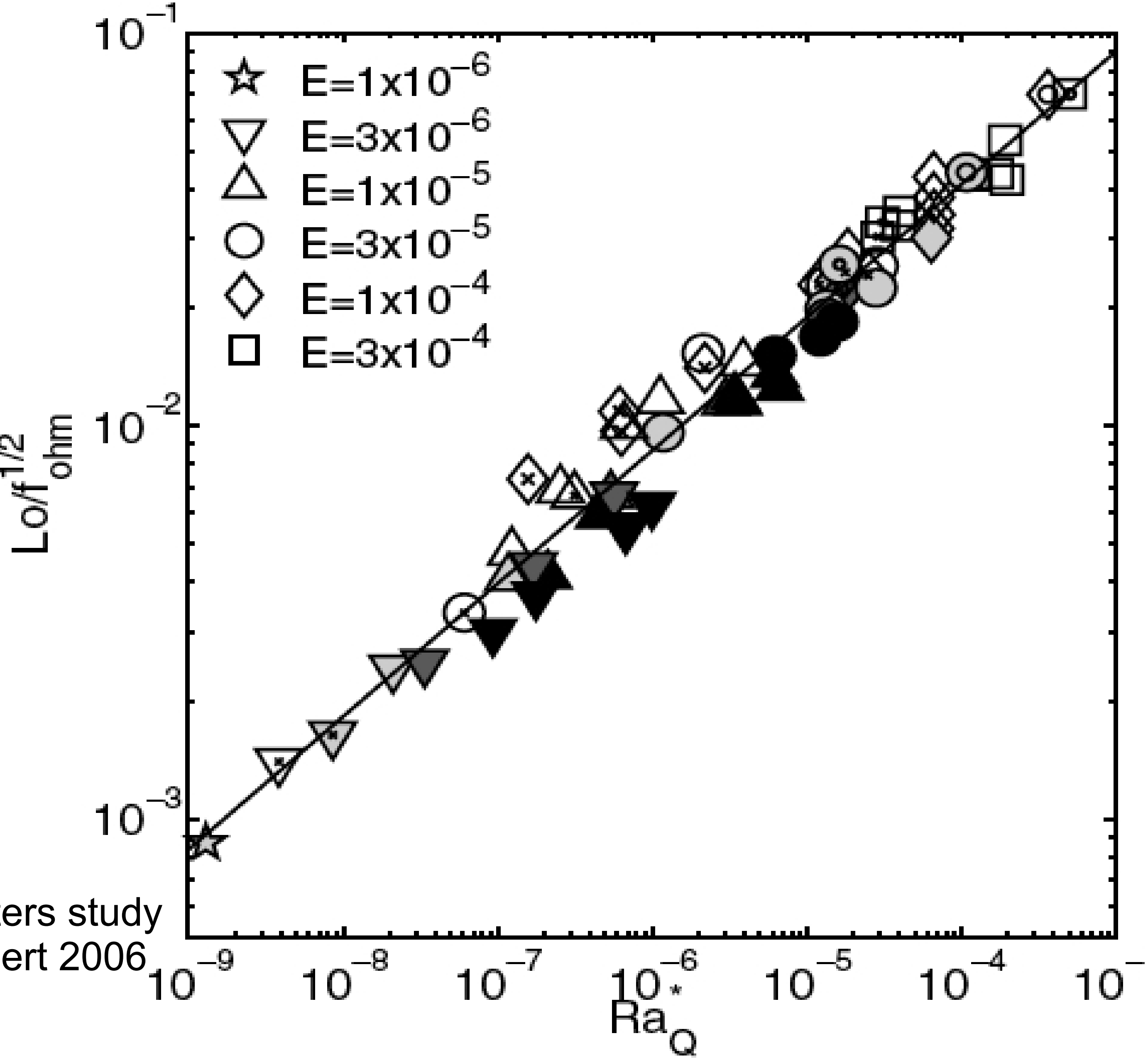
Dynamo branches with SF-SF boundary conditions



Conclusion

- Dynamo mechanism has been highlighted using test-field method (Schrinner et al 2012) and it is useful to understand dynamo bifurcation.
- Subcritical bifurcation obtained for all Ekman numbers.
  - Long period geomag. Evolution.
  - Important reslt for theoretical aspects.
- Distinction between weak and strong field branches is not clear in geodynamo models. Likely different dynamical regimes according to dimensionless numbers and E/Pm and E/Pr
- Scaling laws...

Boussinesq DNSs with rigid B.C. And scaling laws



Extensive parameters study
Christensen & Aubert 2006

Boussinesq simulations are relevant for low mass stars