Dipolar dynamos in stratified systems

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LRA-LERMA
ÉCOLE NORMALE SUPÉRIEURE

Stellar and Planetary Dynamos
Göttingen – May 27, 2015

Laboratoire d'Étude du Rayonnement et de la Matière en Astrophysique

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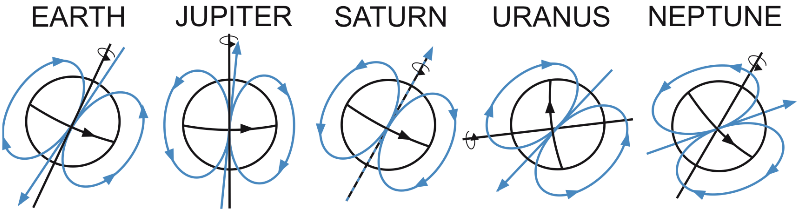
Different magnetospheric configurations



Axisymmetric ($m = 0$) and non-axisymmetric ($m = 1$) configurations

Ridley, V. A., Jovimagnetic secular variation, Doctoral thesis, 2013

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How to model the fluid flow ?

The Convective Approximations

- retain the essential physics with a minimum complexity
- filter out sound waves

The Boussinesq Approximation

$$\nabla \cdot (\mathbf{u}) = 0$$

The Anelastic Approximation

$$\nabla \cdot (\overline{\rho_a}\, \mathbf{u}) = 0$$

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The Boussinesq benchmark

(Christensen et al. 2001)

- homogeneous mass distribution

$$g \propto r$$

The anelastic benchmark

(Jones et al. 2011)

- central mass distribution

$$g \propto 1/r^2$$

The anelastic approximation

Polytropic reference state adiabatically stratified

$$T_a = w(r), \qquad \rho_a = w^n, \qquad P_a = w^{n+1} \tag{1}$$

Resulting system

$$\nabla \cdot (\rho_a \mathbf{V}) = 0 \tag{2}$$

$$D_t \mathbf{V} = -\nabla \left(\frac{P_c}{\rho_a} \right) - \alpha^S S_c \mathbf{g} + \mathbf{F}^\nu / \rho_a \tag{3}$$

$$\rho_a D_t S_c + \nabla \cdot \left(\frac{\mathbf{I}^q}{T_a} \right) = \frac{Q}{T_a} + \mathbf{I}^q \cdot \nabla T_a^{-1} \tag{4}$$

with
- $\mathbf{F}^\nu$ viscous force
- Q viscous heating
- $\mathbf{I}^q$ heat flux

Mean field approach for the heat transfer equation

Solve the equation for average quantities

$$f = \langle f \rangle + f^t \quad (5)$$

Turbulent fluctuations create an entropy flux term

$$\mathbf{I}^{St} = \rho_a \langle S^t \mathbf{V}^t \rangle \propto \nabla \langle S \rangle \quad (6)$$

Strong simplification

- the usual molecular term $\mathbf{I}^q = -k \nabla T$ is then neglected in the heat transfer equation
- $\Longrightarrow$ temperature is removed from the problem !

Drawbacks

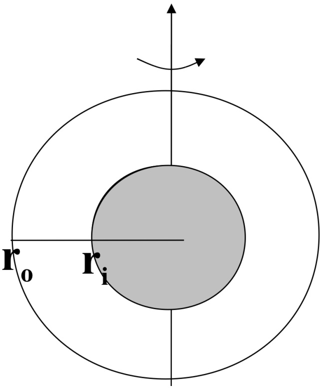
- argument only valid far from the onset of convection
- P2 of thermodynamics not under warranty anymore

MHD equations

Dimensionless system

$$
\begin{aligned}
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} &= Pm\Big[-\frac{1}{E}\nabla\frac{P'}{w^n} + \frac{Pm}{Pr}Ra\frac{S}{r^2}\mathbf{e}_r - \frac{2}{E}\mathbf{e}_z\times\mathbf{v} \\
&\quad + \mathbf{F}^\nu + \frac{1}{E\,w^n}(\nabla\times\mathbf{B})\times\mathbf{B}\Big], && (7)\\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla\times(\mathbf{v}\times\mathbf{B}) + \nabla^2\mathbf{B}, && (8)\\
\frac{\partial S}{\partial t} + \mathbf{v}\cdot\nabla S &= w^{-n-1}\frac{Pm}{Pr}\nabla\cdot\left(w^{n+1}\nabla S\right) \\
&\quad + \frac{Di}{w}\left[E^{-1}w^{-n}(\nabla\times\mathbf{B})^2 + Q^\nu\right], && (9)\\
\nabla\cdot(w^n\mathbf{v}) &= 0, && (10)\\
\nabla\cdot\mathbf{B} &= 0. && (11)
\end{aligned}
$$

A simplified model



Set up

- A perfect gas in a rotating spherical shell with constant
  - ⋆ kinematic viscosity ν
  - ⋆ turbulent entropy diffusivity κ
  - ⋆ magnetic diffusivity η
- Convection is driven by an imposed entropy difference ΔS between the inner and outer spheres

Boundary conditions

- fixed entropy
- stress-free b. c. for the velocity field
- insulating b. c. for the magnetic field

Systematic parameter study (~ 250 models)

Seven control parameters

Rayleigh number	Ra	$\frac{GMd\Delta S}{\nu\kappa c_p}$	$\mathcal{O}(10^6)$
magnetic Prandtl number	Pm	ν/η	$\mathcal{O}(1)$
Prandtl number	Pr	ν/κ	1
Ekman number	E	$\nu/(\Omega d^2)$	10^{-4}
aspect ratio	χ	r_i/r_o	0.35
polytropic index	n	$1/(\gamma-1)$	2
number of density scale heights	N_ϱ	$\ln(\varrho_i/\varrho_o)$	≤ 3

The anelastic version of PaRoDy reproduces the anelastic dynamo benchmark (Jones et al. 2011).

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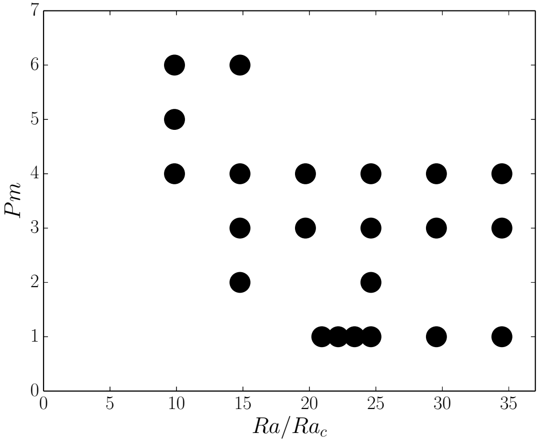
Stability domain of the dipolar branch

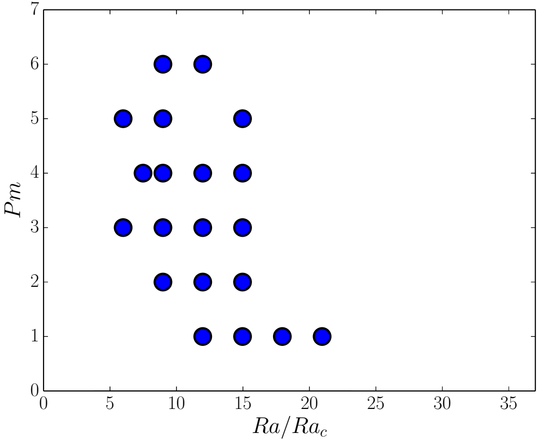
$N_\varrho = 0.1$



Dipolar dynamos as a function of Ra/Ra_c and Pm

Stability domain of the dipolar branch

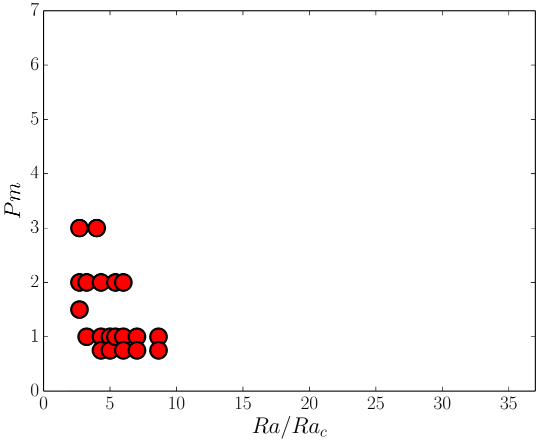
$N_\varrho = 0.5$



Dipolar dynamos as a function of Ra/Ra_c and Pm

Stability domain of the dipolar branch

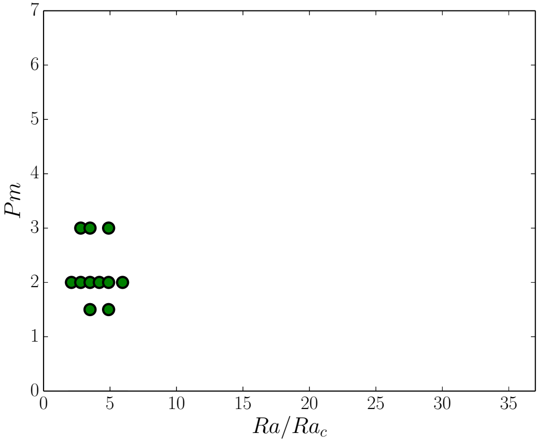
$N_\varrho = 1.5$



Dipolar dynamos as a function of Ra/Ra_c and Pm

Stability domain of the dipolar branch

$N_\varrho = 2.0$



Dipolar dynamos as a function of Ra/Ra_c and Pm

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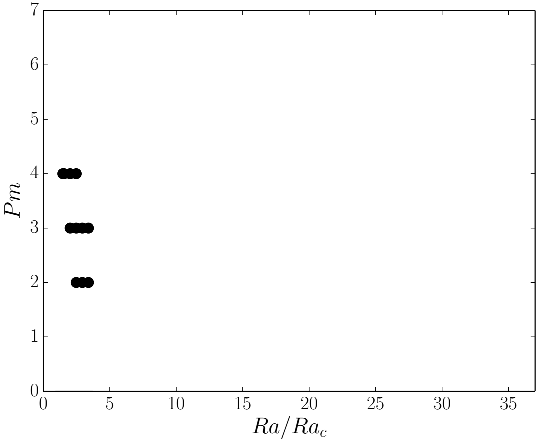
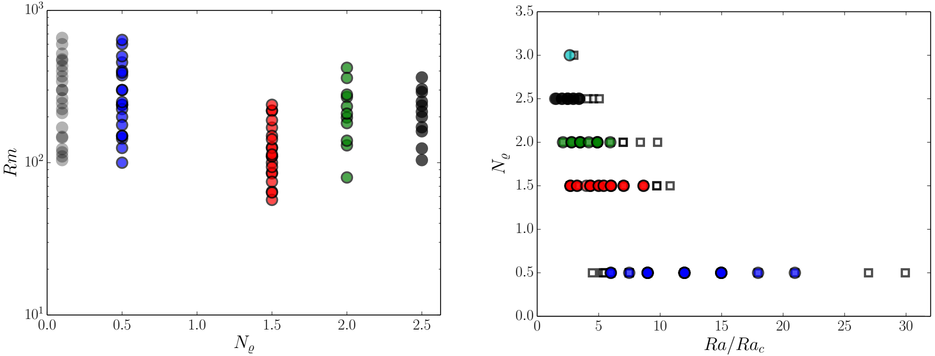


Dipolar dynamos as a function of Ra/Ra_c and Pm

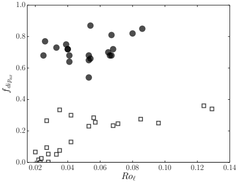
Critical magnetic Reynolds number

$$\forall N_\varrho , \qquad Rm_c \sim 10^2$$

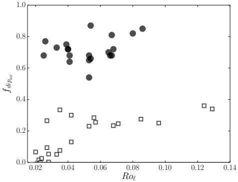


Role of inertia: local Rossby number

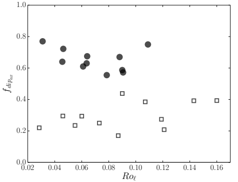
$N_\varrho = 0.5$



$N_\varrho = 1.5$



$N_\varrho = 2.0$



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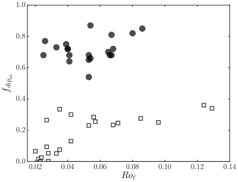

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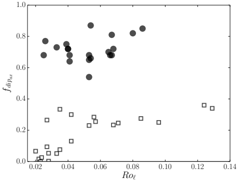

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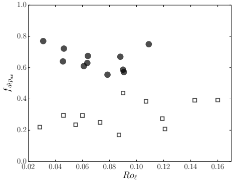
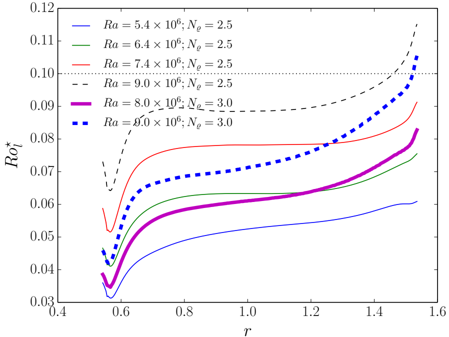


$Ro_\ell = Ro_c \, \ell_c / \pi$

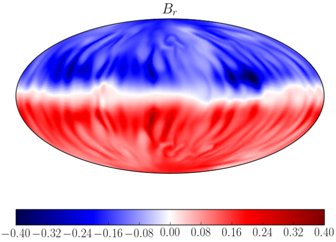
as a function of radius for dipolar (solid lines) and multipolar (dashed lines) dynamos at ($N_\varrho = 2.5, Pm = 2$) (thin lines) and ($N_\varrho = 3, Pm = 4$) (thick lines).

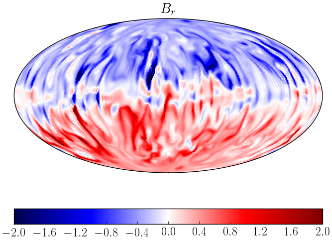


Snapshots

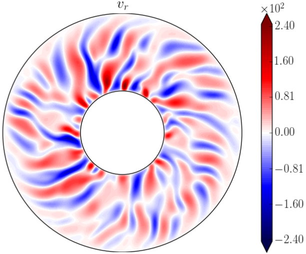
$N_\varrho = 1.5, Pm = 0.75, Ra/Ra_c = 5$ | $N_\varrho = 2.5, Pm = 2, Ra/Ra_c = 3.4$

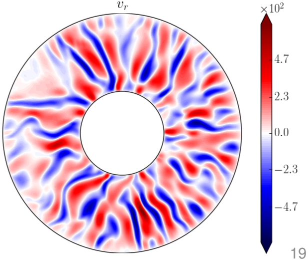
$B_r(r = r_o)$





V_r





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Results

- Dipolar solutions can be observed at high N_ϱ, provided high enough *Pm* are considered.
- The critical *Rm* of the dipolar branch seems scarcely affected by the increase of N_ϱ.
- The higher N_ϱ, the faster is reached the critical Ro_ℓ above which inertia causes the collapse of the dipole. This explains why dipolar dynamos become confined in the parameter space.
- The scarcity of dipolar solutions for substantial N_ϱ would thus rather come from the restriction of the parameter space being currently explored (because of computational limitations), rather than an intrinsic modification of the dynamo mechanisms.

References

- Schrinner, Petitdemange, Raynaud & Dormy, 2014, A&A, 564, A78
- Raynaud, Petitdemange & Dormy, 2014, A&A, 567, A107
- Raynaud, Petitdemange & Dormy, 2015, MNRAS, 448:2055–2065