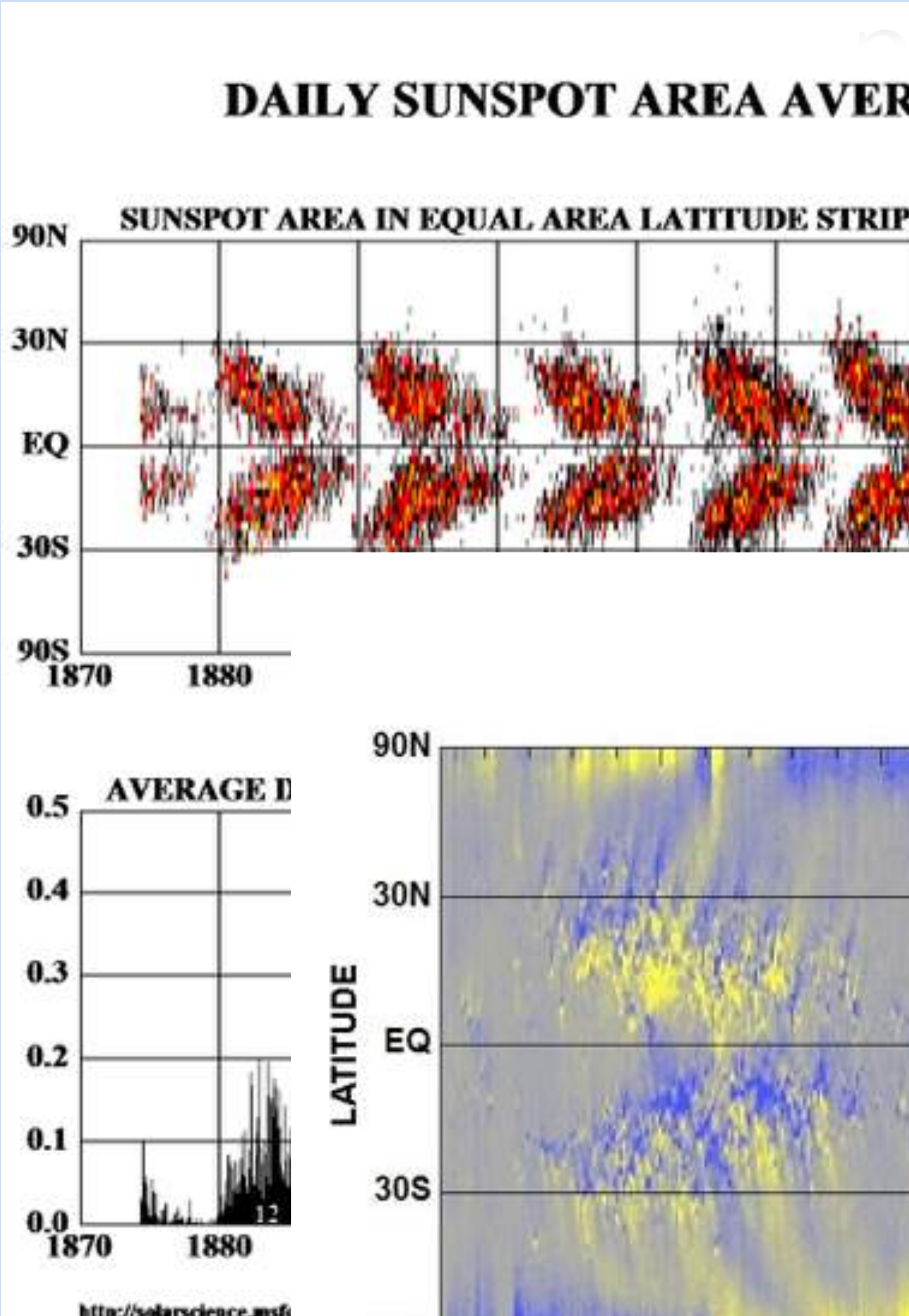
The Understanding the equatorward migration of the Sun's magnetic field

Jörn Warnecke

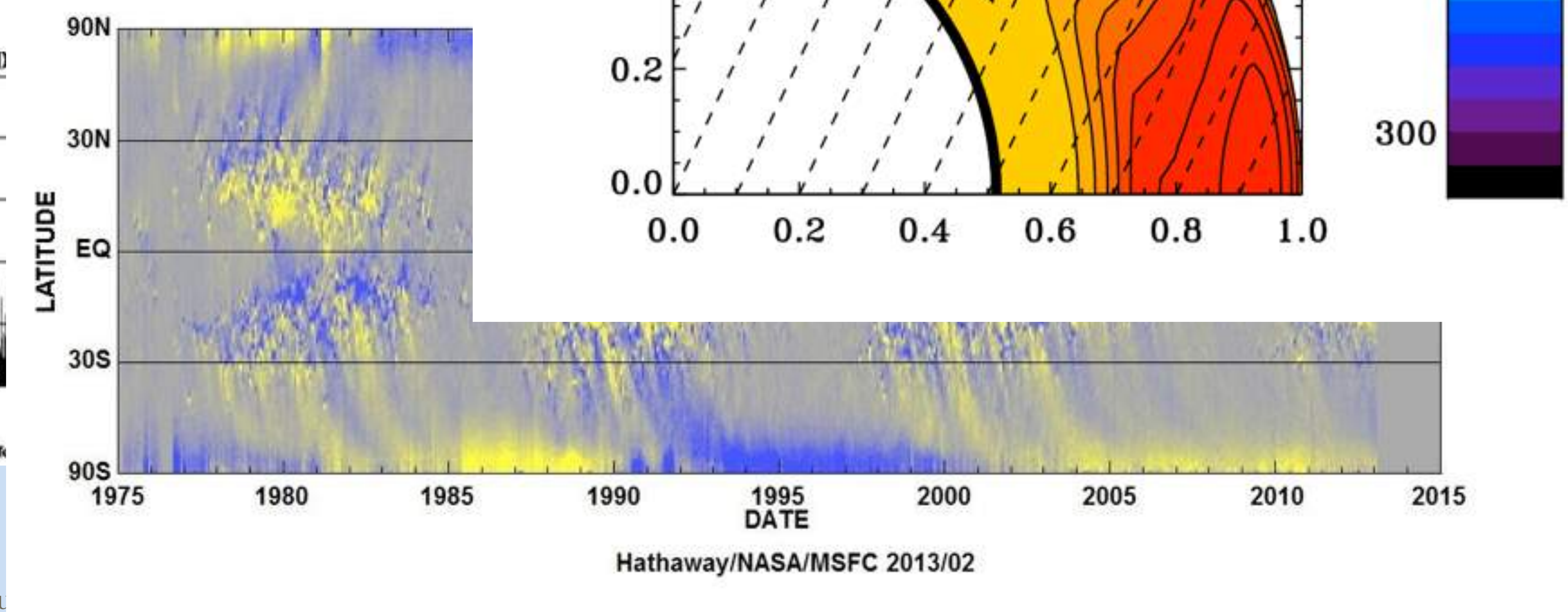
Max Planck Institute
for Solar System Research

Axel Brandenburg, Nordita
Petri J. Käpylä, Helsinki University
Maarit J. Käpylä, Aalto University

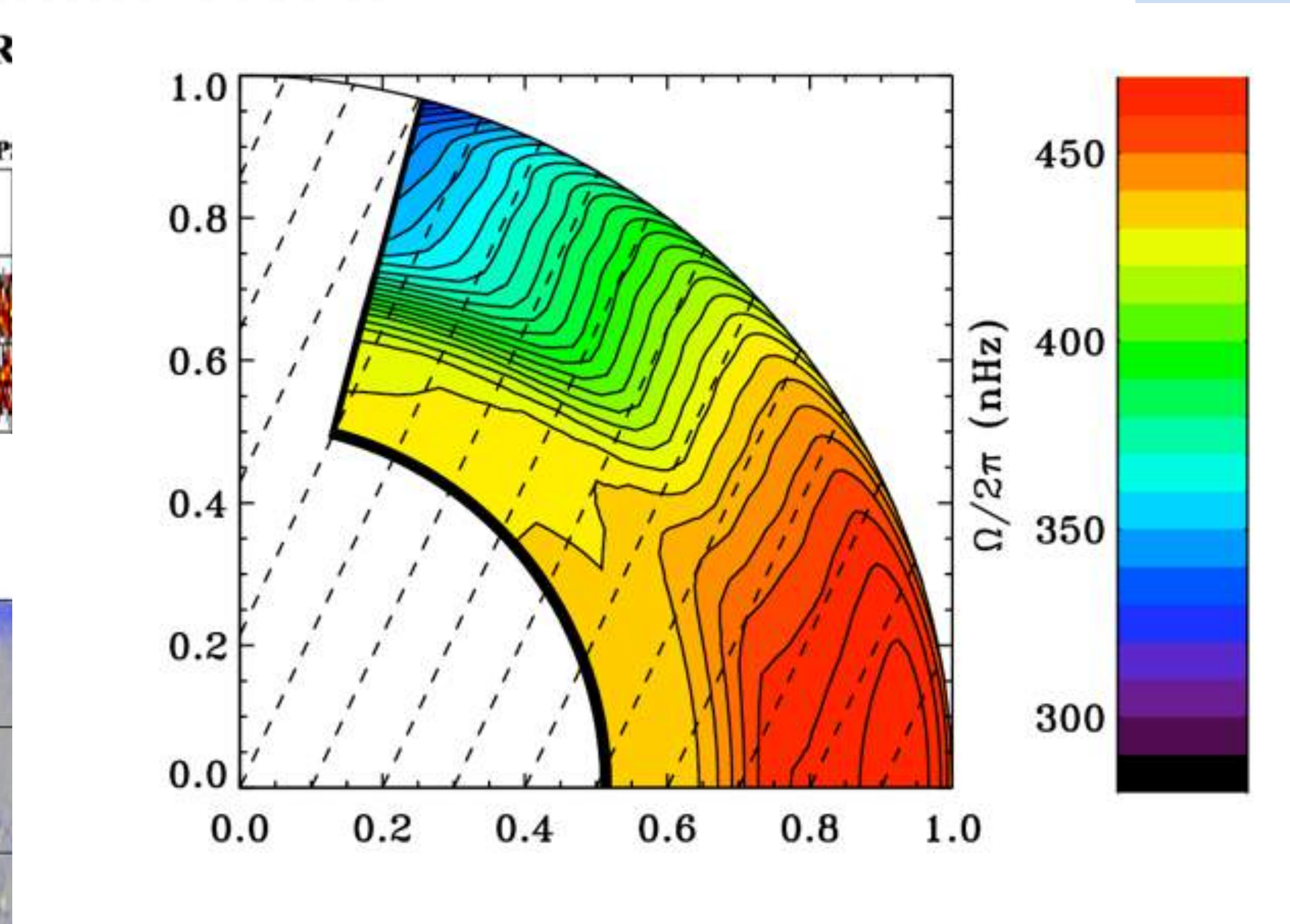
Solar Cycle



DAILY SUNSPOT AREA AVER
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIP
90N
30N
EQ
30S
90S
1870
1880
AVERAGE D
0.5
0.4
0.3
0.2
0.1
0.0
1870
1880
http://solarscience.msf



90N
30N
EQ
30S
90S
LATITUDE
1975
1980
1985
1990
1995
2000
2005
2010
2015
DATE
Hathaway/NASA/MSFC 2013/02



1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω/2π (nHz)
450
400
350
300

Global convective dynamo simulations

$$\frac{\partial A}{\partial t} = u \times B + \eta \nabla^2 A$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot u$$

$$\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho}\left(J \times B - \nabla p + \nabla \cdot 2\nu\rho S\right)$$

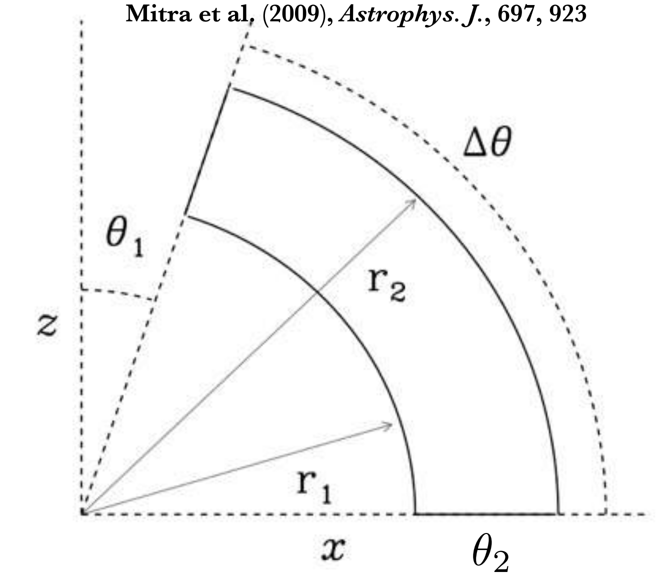
$$T\frac{Ds}{Dt} = \frac{1}{\rho}\nabla \cdot (K\nabla T + \chi_t \rho T \nabla s) + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma_{\mathrm{cool}}(r),$$

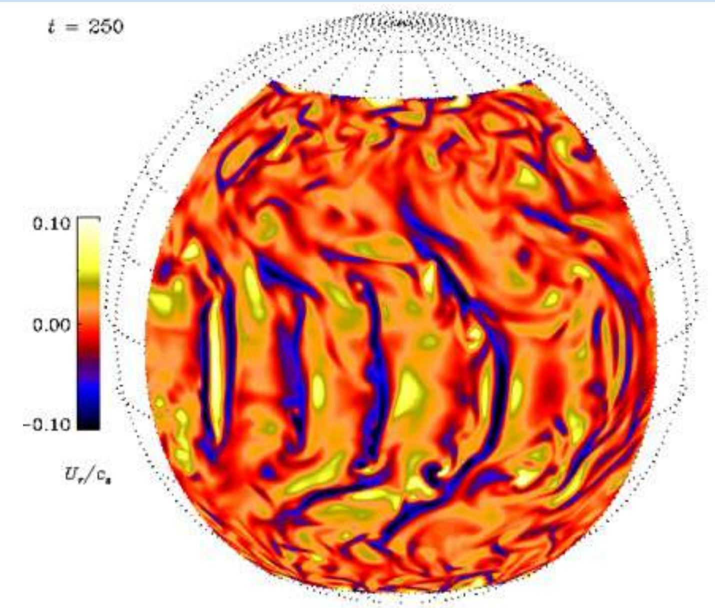
- high-order finite-difference code
- scales up efficiently to over 60.000 cores
- compressible MHD

http://pencil-code.google.com/

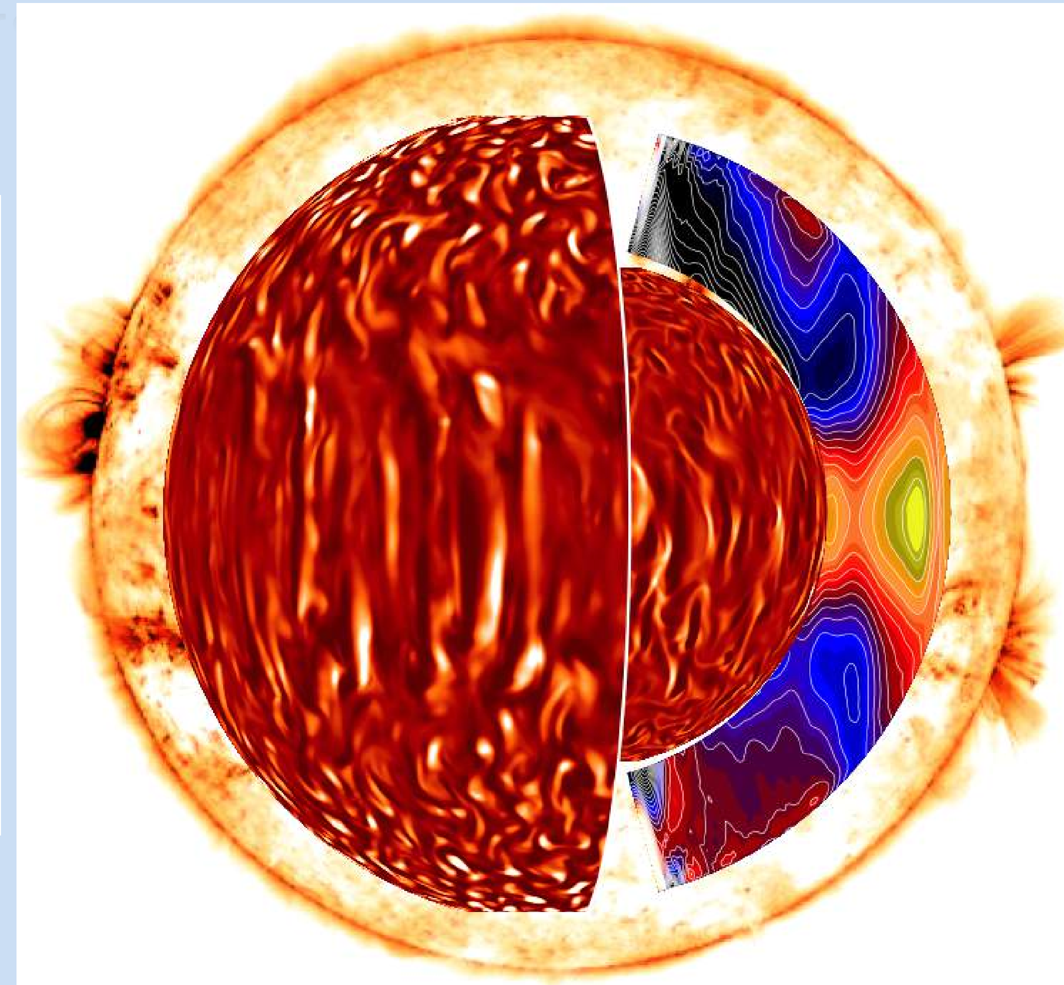
Global convective dynamo simulations

Mitra et al, (2009), *Astrophys. J.*, 697, 923





Käpylä et al. (2010b), *Astron. Nachr.*, 331, 73

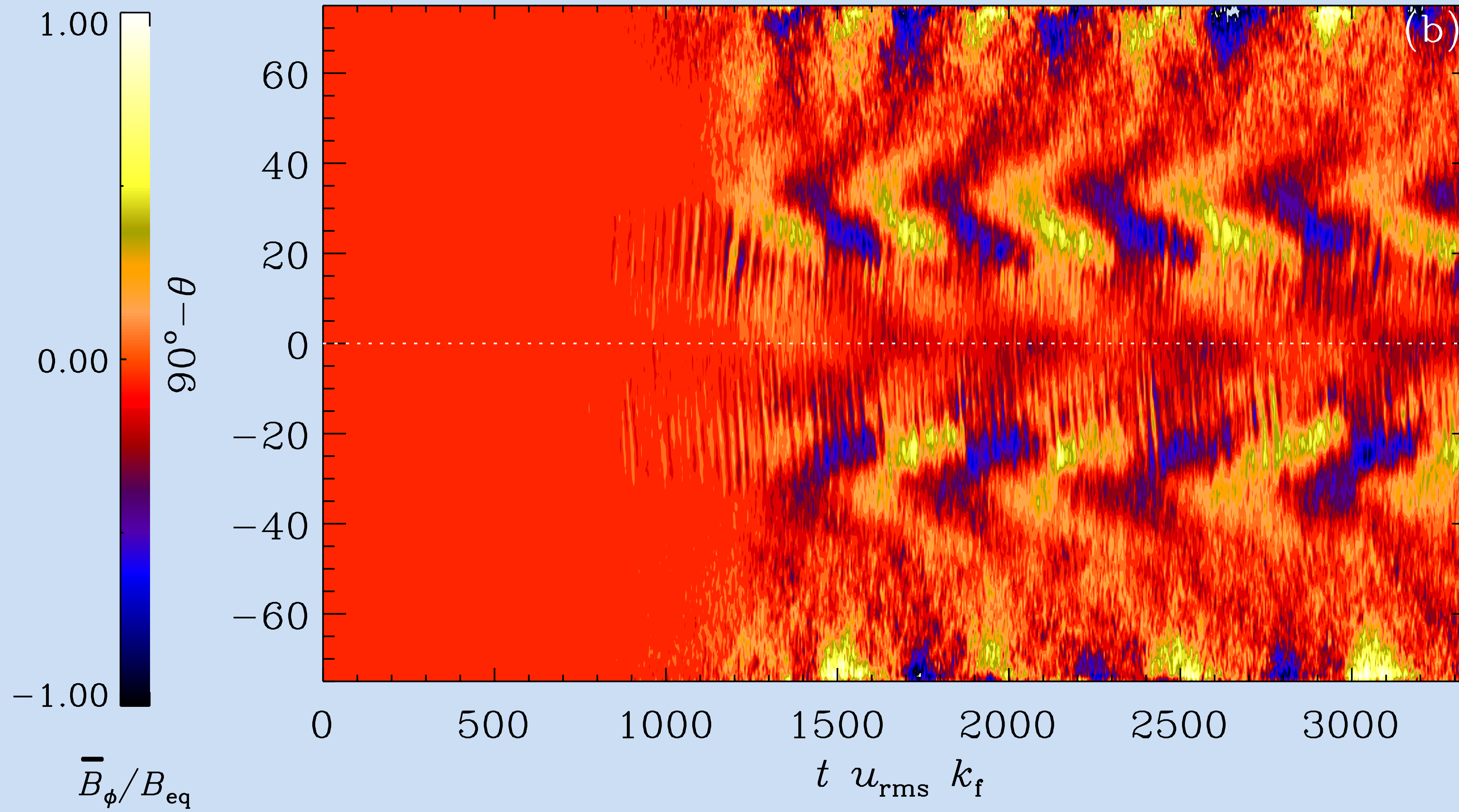


$0.7R < r < R$ $\theta_1 < \theta < \theta_2$ $0 < \phi < \Delta\phi$ $k_{\rm f} = 2\pi/\Delta R$

We model a spherical sector (`wedge') where only parts of the latitudinal and longitudinal extents are taken into account.

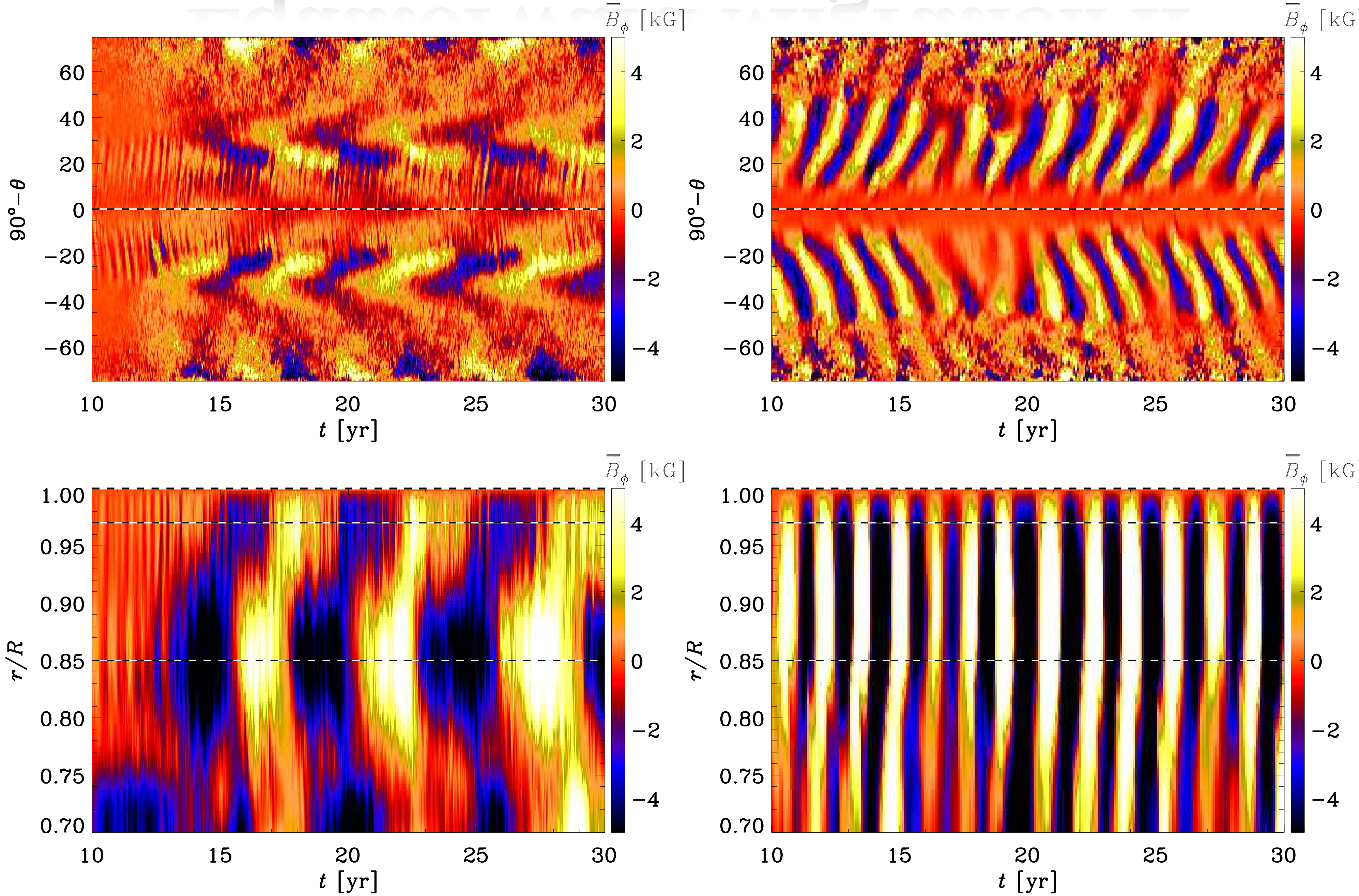
Normal field condition for *B* at the outer radial boundary and perfect conductor at all other boundaries. Impenetrable stress-free boundaries on all boundaries.

Equatorward Migration I



Käpylä, Mantere & Brandenburg 2012 (ApJL 755, L22)

Equatorward Migration II



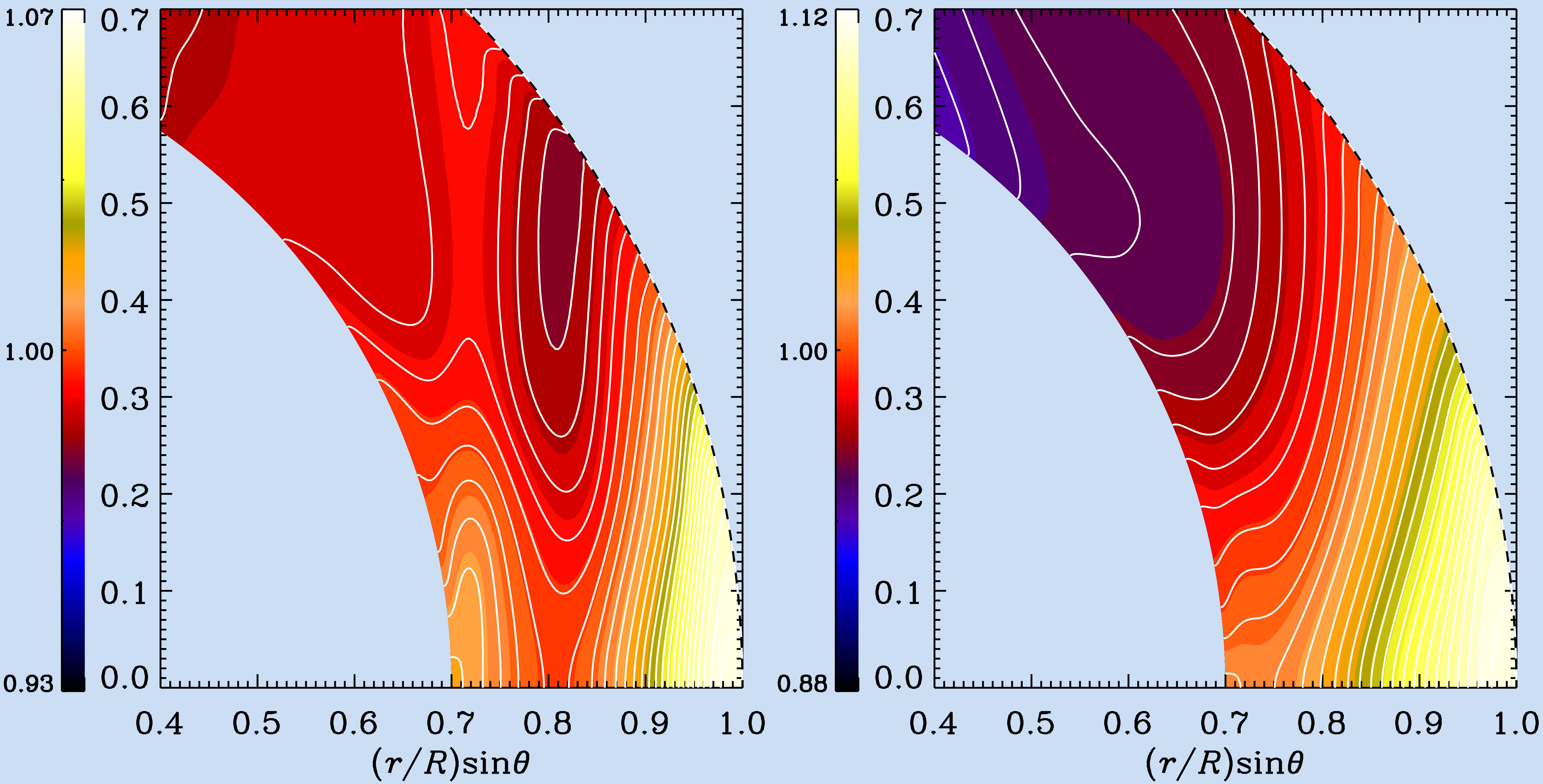
Pr=ν/χ=2.5

Pm=ν/η=1

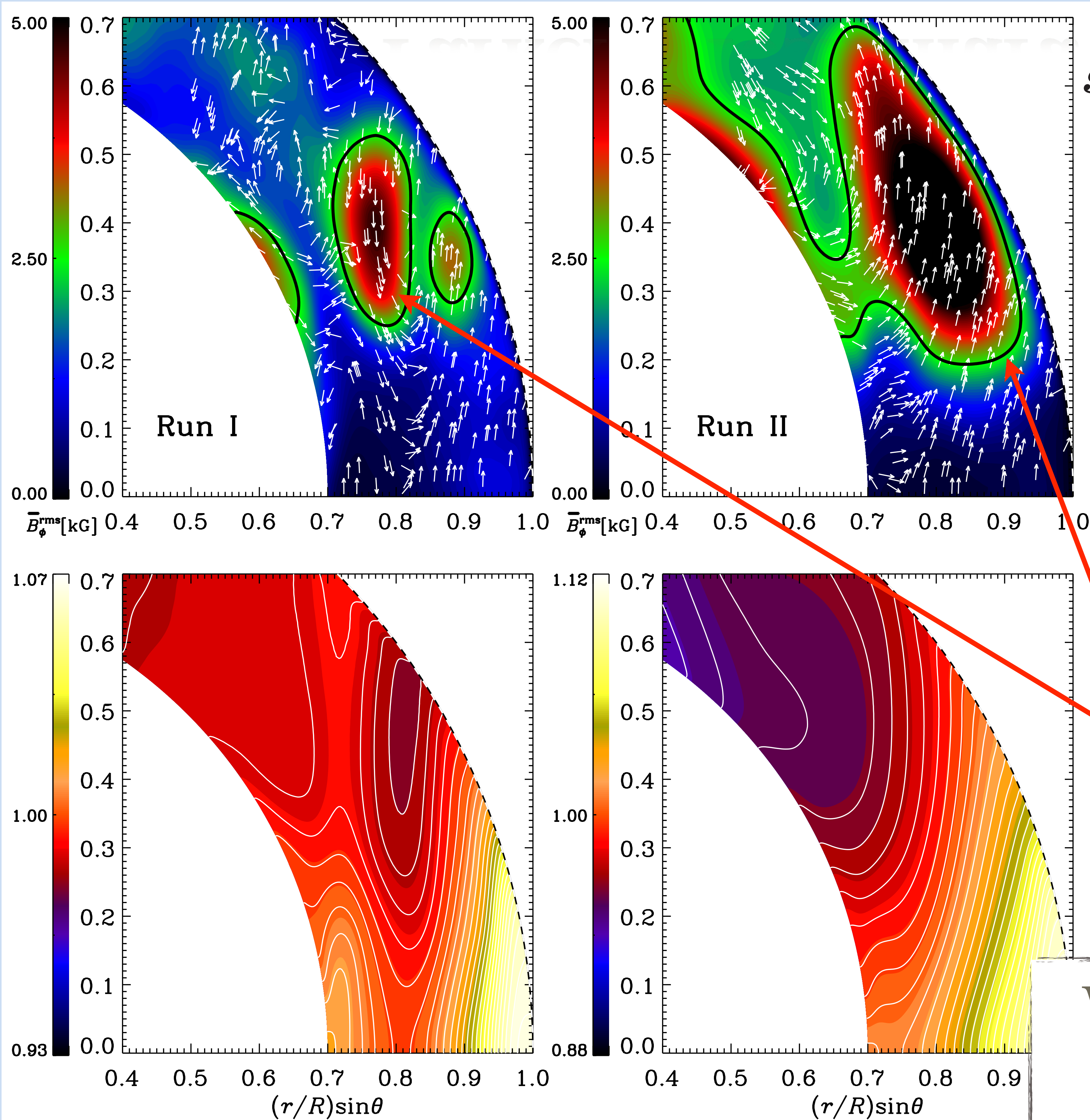
Pr=0.5

Pm=0.5

Differential rotation



Parker—Yoshimura—Rule



$$\boldsymbol{s}_{\rm mig}(r,\theta) = -\alpha \hat{\boldsymbol{e}}_\phi \times \boldsymbol{\nabla}\Omega,$$

Parker 1955

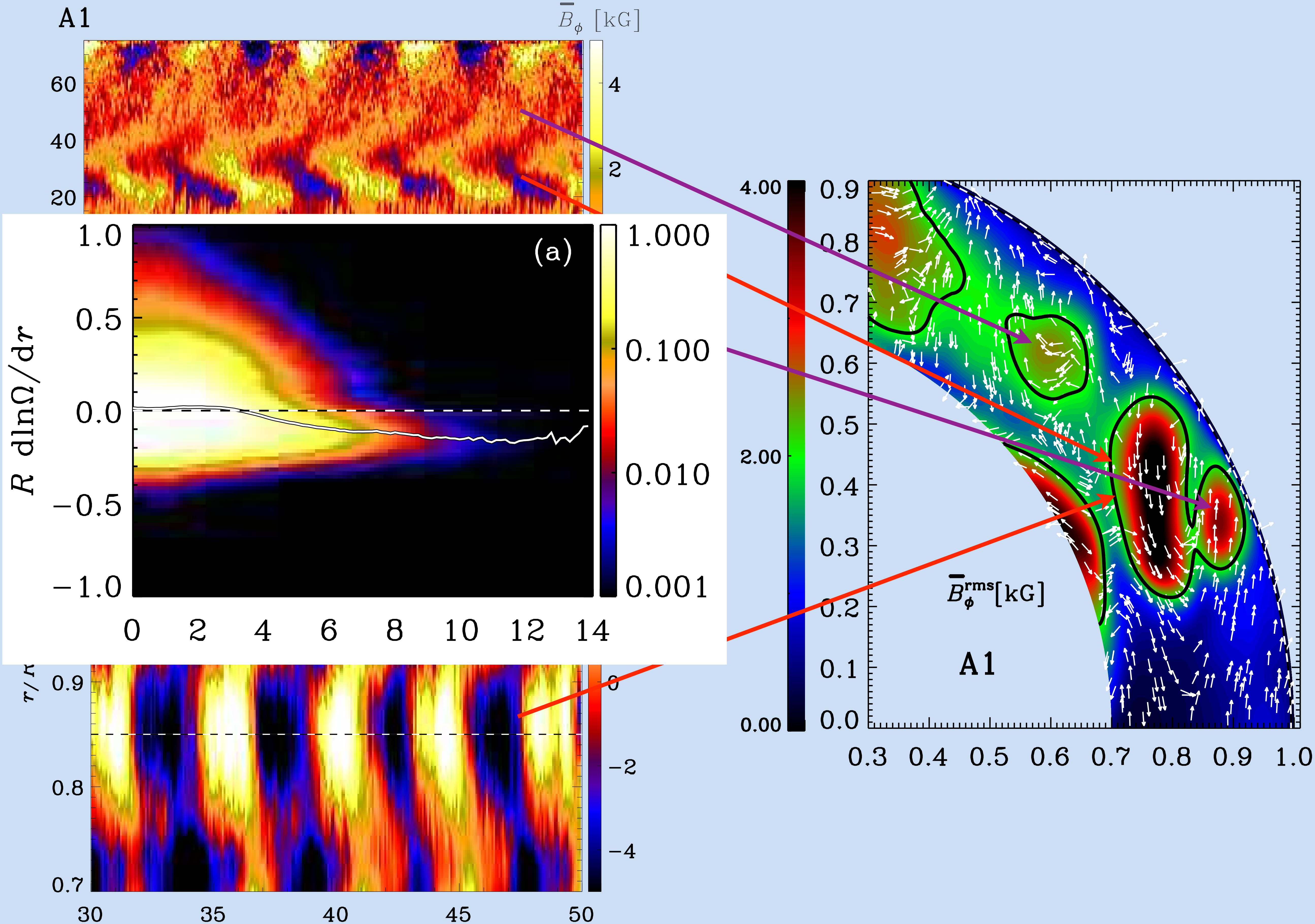
Yoshimura 1975

$$\alpha = \frac{\tau_{\rm c}}{3}\left(-\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \frac{\overline{\boldsymbol{j}\cdot\boldsymbol{b}}}{\overline{\rho}}\right)$$

Pouquet et al. 1976

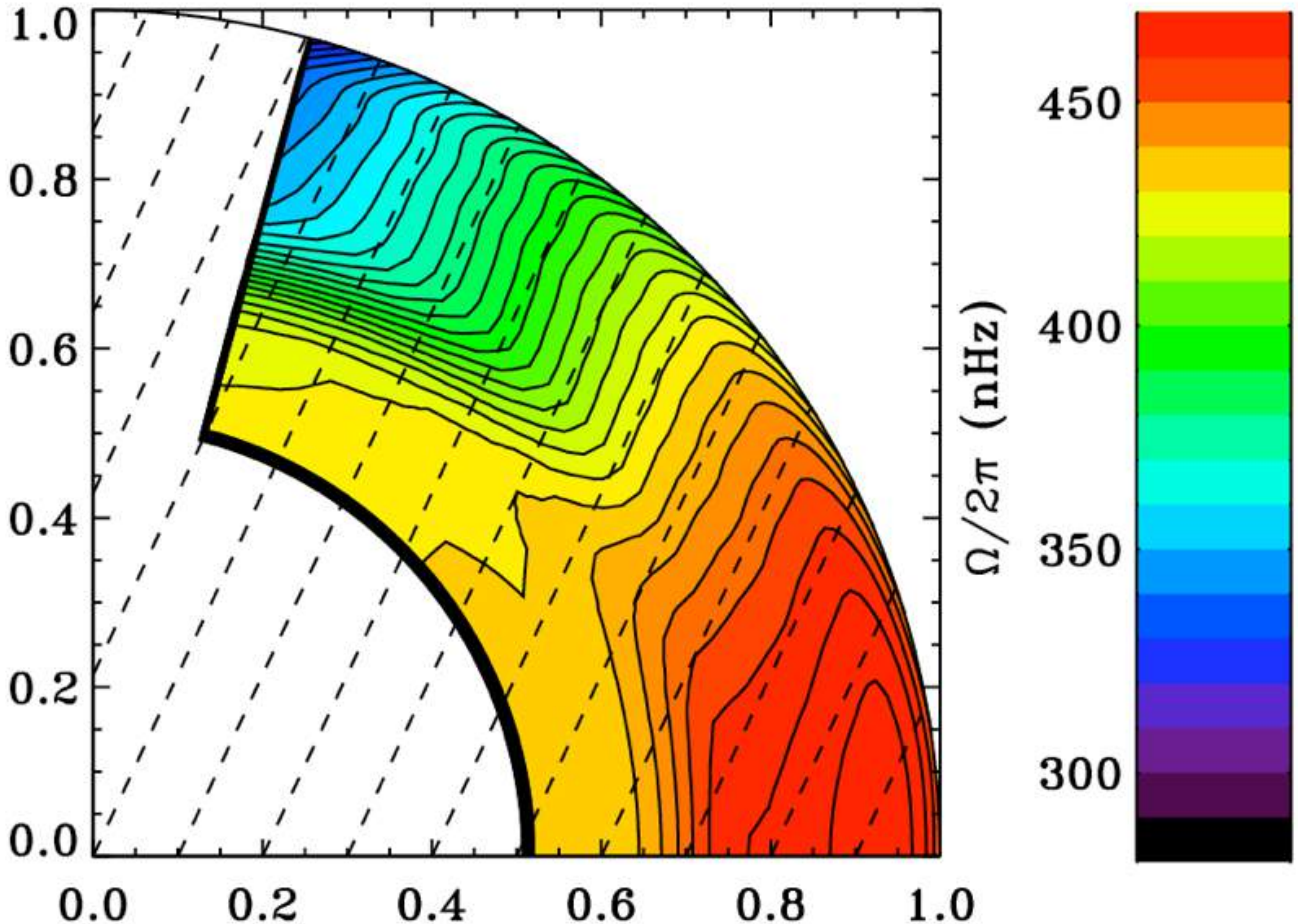
Strong toroidal field

Warnecke et al. 2014 (ApJL 796, L12)


A1
B̄_φ [kG]
60
40
20
4
2
1.0
0.5
0.0
−0.5
−1.0
R dlnΩ/dr
(a)
1.000
0.100
0.010
0.001
0 2 4 6 8 10 12 14
0.9
0.8
0.7
r/R
0
−2
−4
30 35 40 45 50
4.00
2.00
0.00
0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
B̄_φ^rms [kG]
A1

Propagation direction of mean toroidal magnetic field can be entirely explain by the Parker—Yoshimura—Rule

(Warnecke et al. 2014, 2015a)


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Ω/2π (nHz)
450
400
350
300

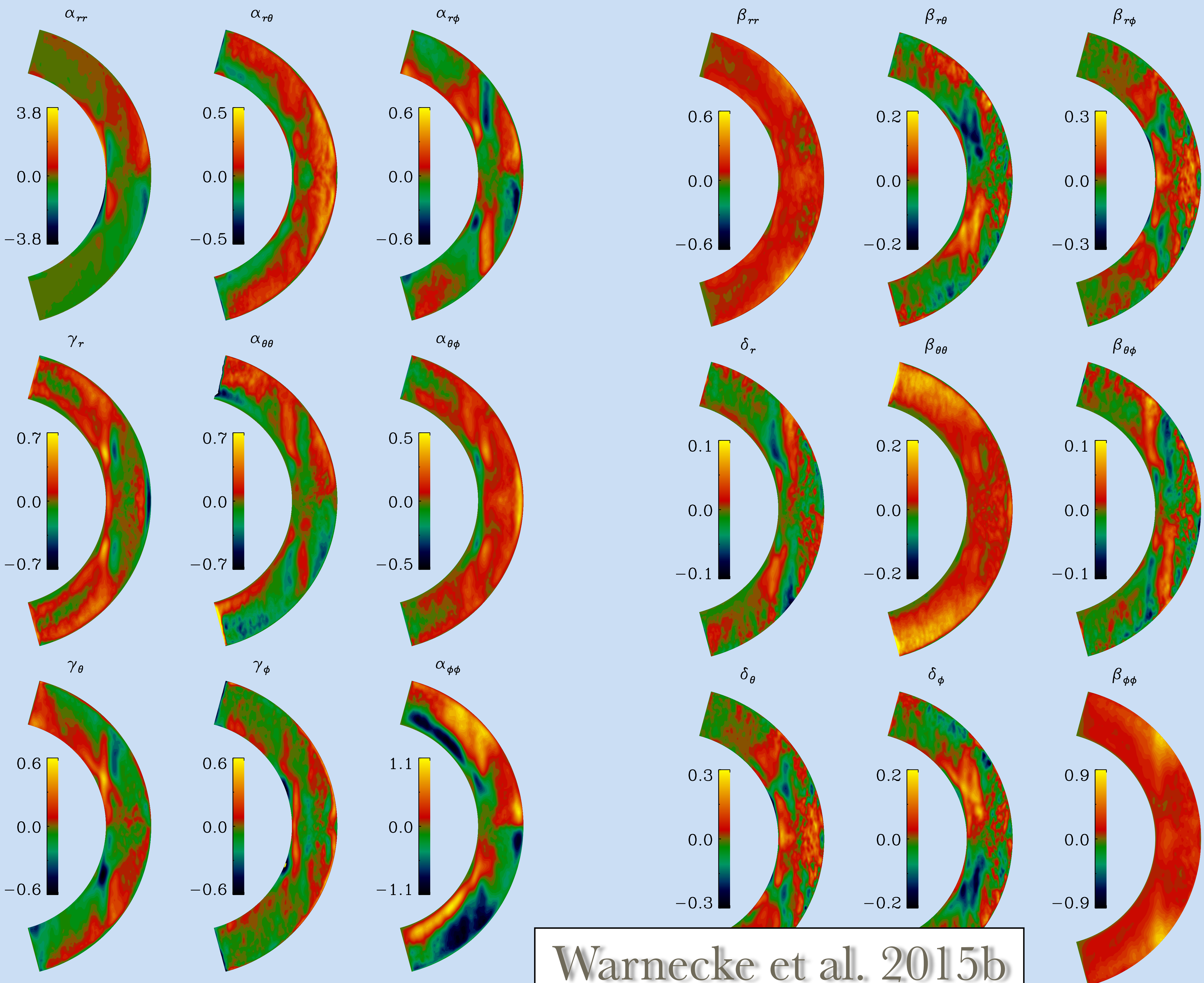
Test-field method

Determine the turbulent transport coefficients from global convective simulations

Schrinner et al. 2005, 2007

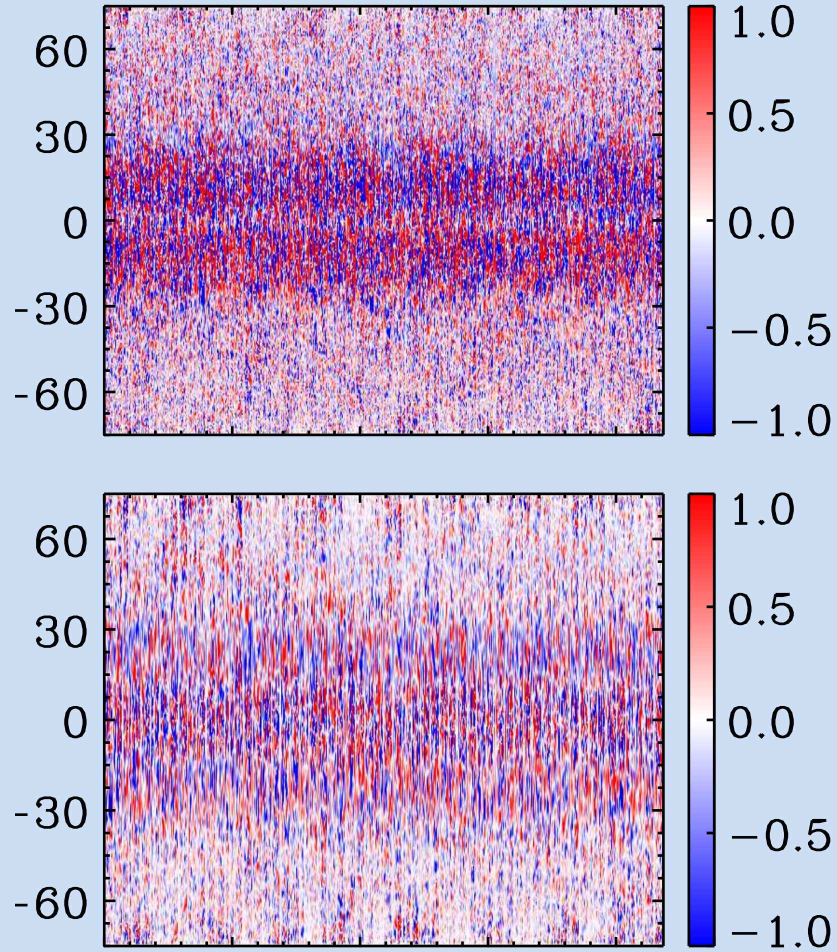
$$\overline{\mathcal{E}} = \overline{u' \times b'} = \alpha \overline{B} + \gamma \times \overline{B} + \beta \nabla \times \overline{B} + \delta \times (\nabla \times \overline{B}) + \kappa \nabla \overline{B}$$

Solving 27 equations for b',
with 27 independent test-fields

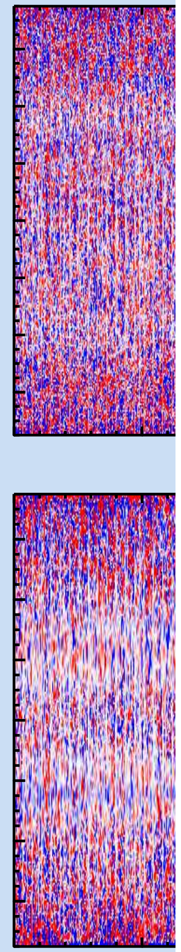


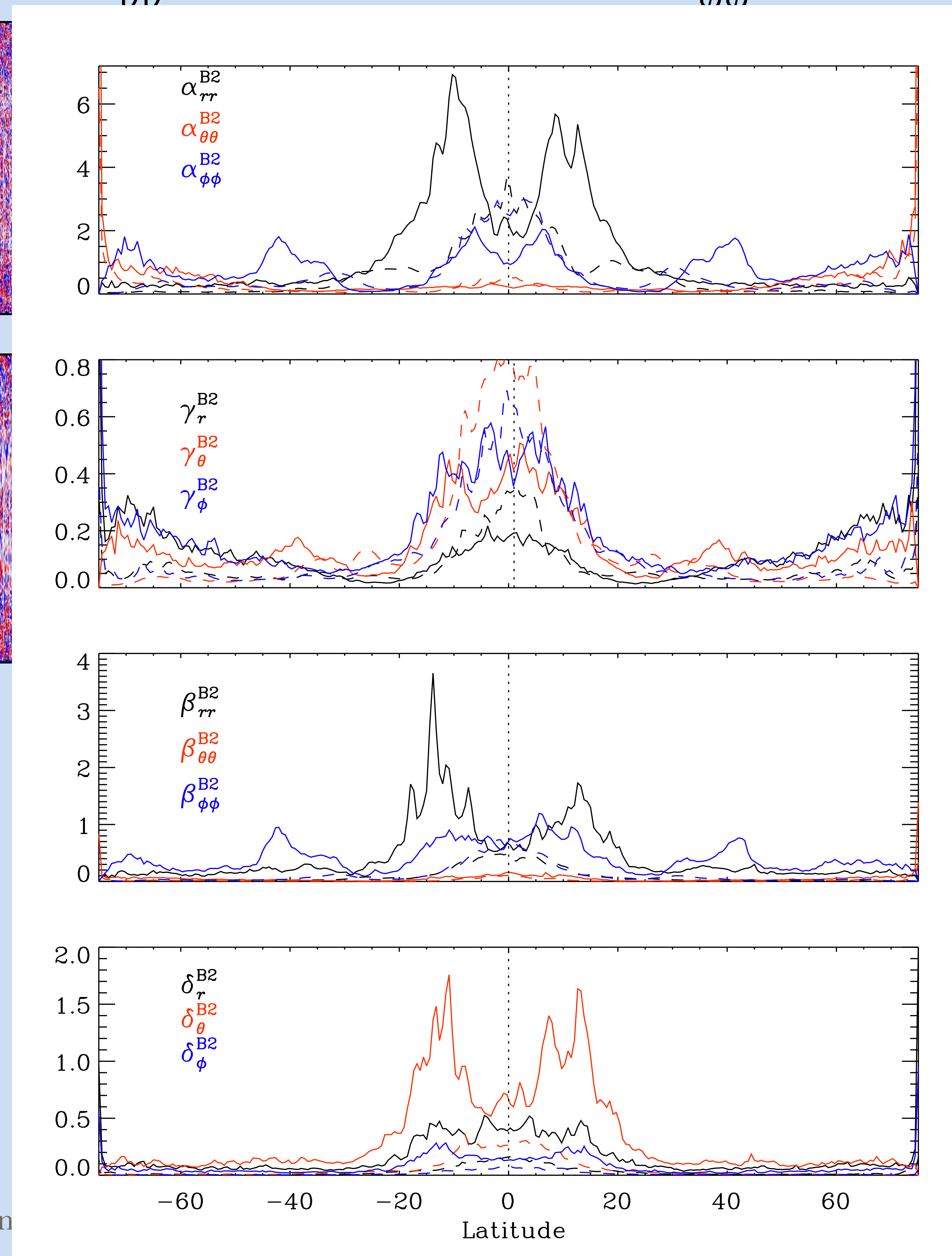
Warnecke et al. 2015b
(in preparation)

α_{rr} $\alpha_{\theta\theta}$ $\alpha_{\phi\phi}$



time



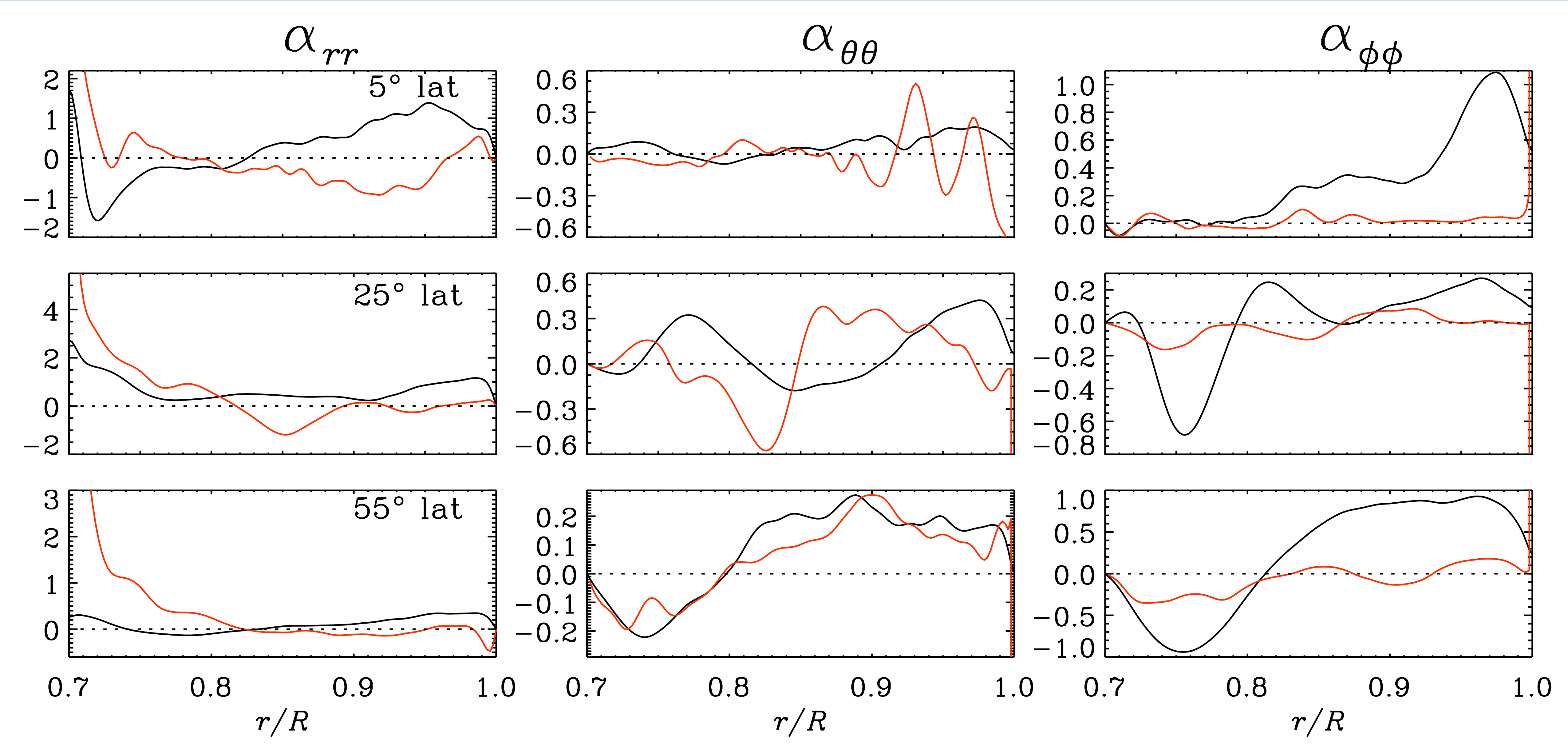


$$\alpha_{ii}^{B2} = \frac{\langle (\alpha_{ii}\overline{B}_\phi)^2 \rangle_t}{\langle \overline{B}_\phi^2 \rangle_t}$$

Test-field method vs. Singular value decomposition (SVD)

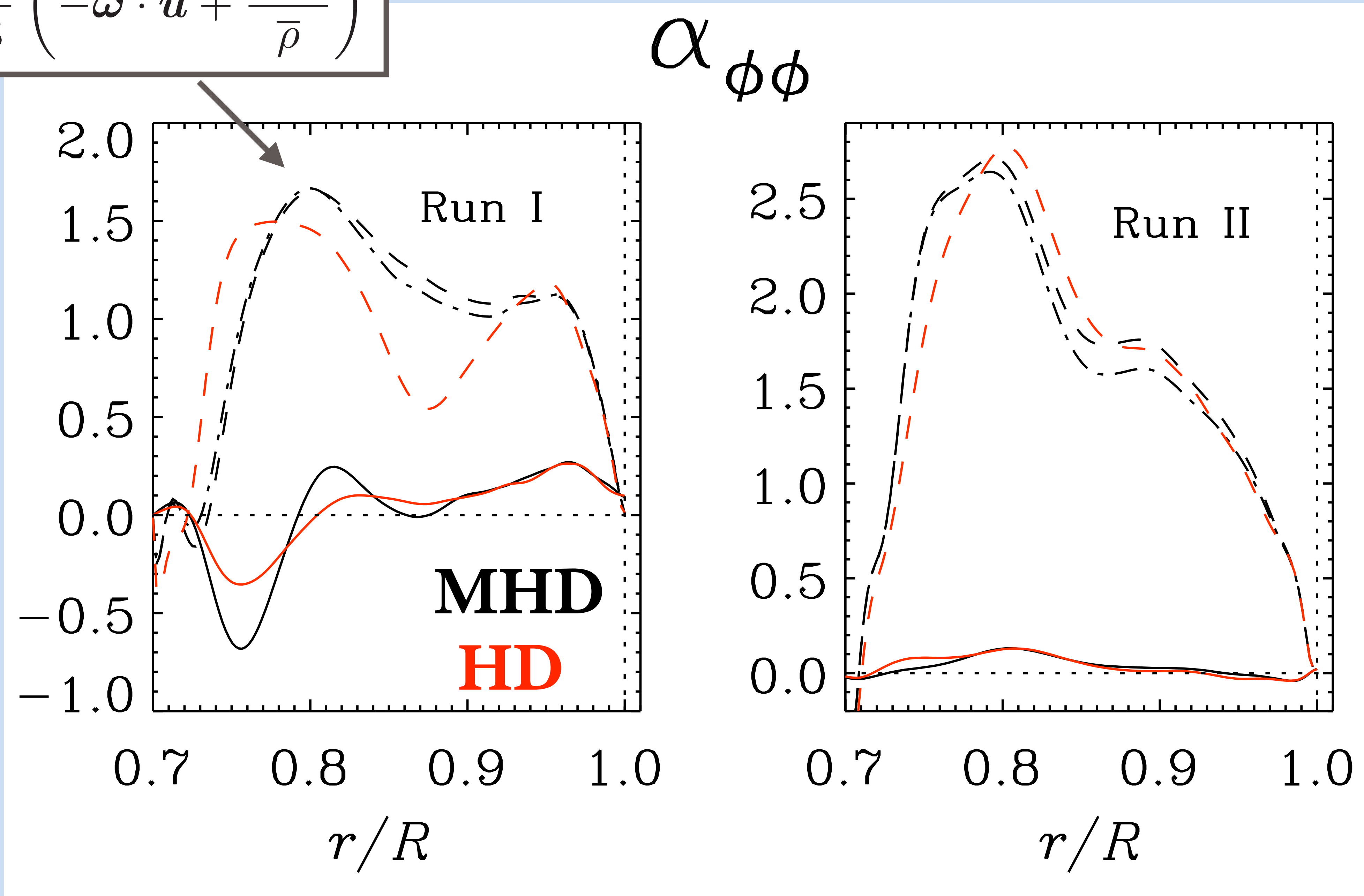
$$\overline{\mathcal{E}} = \overline{u' \times b'} = \alpha \overline{B} + \gamma \times \overline{B} + \beta \nabla \times \overline{B} + \delta \times (\nabla \times \overline{B}) + \kappa \nabla \overline{B}$$

(Racine et al. 2011, Augustson et al. 2014)



Parker Wave ?

$$\alpha = \frac{\tau_c}{3}\left(-\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \frac{\overline{\boldsymbol{j}\cdot\boldsymbol{b}}}{\overline{\rho}}\right)$$



Conclusions

- Equatorward propagation in simulation are related to the negative shear.
- Migration of mean magnetic field can be entirely explained by an alpha-omega-dynamo wave
- Parker-Yoshimura-Rule works!
- Test-field results for solar dynamo simulations.
- Fluctuations correlate with the magnetic cycle.
- SVD does not agree with test-field results.

Please give back your badges before you leave!